Physics-Constrained Neural Networks for Electromagnetic Surrogate Modeling

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***Abstract*—This paper presents a neural network-powered framework for accelerating electromagnetic simulations for early-stage antenna design. The framework addresses the computational bottlenecks associated with conventional surrogate models and interpolation techniques by introducing compact, physics-compliant surrogate models capable of predicting spherical wave expansion (SWE) coefficients and scattering parameters with high accuracy and in real-time. By leveraging the SWE representation, the framework ensures Maxwell-consistent predictions while providing memory-efficient field reconstruction at any point within the region of validity. The capabilities of the proposed framework are demonstrated through a case study involving a multi-layer patch antenna designed for Ka-band operation. The study highlights the framework's ability to deliver accurate, real-time predictions of electromagnetic fields and scattering parameters across a multi-parameter design space, showcasing its potential as a scalable and efficient tool for antenna design engineers.**

***Index Terms*—Machine Learning, Antenna Design, Computational Electromagnetics, Surrogate Modelling, Deep Learning**

I. Introduction

Designing advanced antenna systems typically involves many design parameters and requires high-fidelity electromagnetic (EM) simulations to validate performance and ensure compliance with stringent specifications. The complexity of the design process often makes it inherently exploratory and iterative, involving extensive parameter studies. A challenge is that the computational overhead of repeated EM simulations frequently constrains the scope and number of these studies. Specifically, every adjustment to an antenna model's geometrical or electrical parameters requires the simulations to be recomputed from scratch. This results in significant delays, restricts timely feedback, and slows the iterative design process, thereby limiting decision-making and design exploration.

Traditional workflows have sought to address these challenges through surrogate modeling [1]. Conventional approaches, such as polynomial fitting, kriging, or radial basis functions [2], can approximate performance in low-dimensional parameter spaces. However, these methods are fundamentally limited by the curse of dimensionality, which may lead to an exponential increase in the required number of samples as the parameter space expands to meet accuracy requirements. This can make such methods impractical or even infeasible for complex, modern antenna designs.

To address these challenges, this work introduces a novel neural network-powered framework that provides low-memory, high-accuracy, and physics-compliant surrogate models for antenna design. Concretely, the key contributions of this work are twofold:

1) The proposed framework simultaneously predicts spherical wave expansion (SWE) coefficients and scattering (S)-parameters, unlike conventional approaches that directly predict EM fields. Predicting SWE coefficients offers several distinct advantages over predicting EM fields: a) they ensure that Maxwell's equations are always satisfied, b) their compressed representation of EM fields significantly reduces memory requirements, and c) due to them being independent of the choice of field sampling points, they enable efficient reconstruction of EM fields at any point in space within the region of validity.
2) The framework introduces a class of specialized complex-valued neural network architectures. These architectures are explicitly designed to capture both phase and amplitude information, which is crucial for EM data. Compared to traditional real-valued networks that struggle with accurately representing phase-dependent data, complex-valued networks leverage the inherent properties of complex numbers. This design ensures superior accuracy and efficiency, particularly for predicting the complex-valued SWE coefficients and S-parameters.

To demonstrate the capabilities of the proposed framework, we present a case study involving a multi-layer patch antenna designed for Ka-band operation. This case study highlights the framework's ability to accurately model EM fields and S-parameters in real-time across a seven-parameter design space. By leveraging the SWE representation, the framework enables field predictions at any point within the region of validity, offering a clear advantage over traditional field-predicting methods in terms of accuracy and flexibility. This demonstrates the possibility for faster and far more interactive workflows in early-stage antenna design, ultimately shortening design cycles and broadening the scope of the exploratory design phase.

II. Neural Network Framework

To achieve accurate, efficient, and physically consistent surrogate modeling of antennas, we have created a complex-valued neural network (NN) that predicts S-parameters and spherical wave expansion coefficients.

A. Spherical Wave Expansion

Spherical wave expansion (SWE) is a way of representing solutions to Maxwell's equations [3]. The SWE basis functions form a complete basis for describing outward-propagating EM fields. One of their most valuable properties is that any linear combination of them will automatically satisfy Maxwell's equations. Instead of training our NN model to predict EM field values directly, we make it predict coefficients for these basis functions. The EM field corresponding to a set of SWE coefficients can then be obtained by computing

$$\boldsymbol{E}(r,\theta,\varphi) = k\sqrt{\zeta}\sum_i Q_i \boldsymbol{F}_i(r,\theta,\varphi), \tag{1}$$

where $k = 2\pi/\lambda$ is the angular wavenumber, ζ is the impedance of free space, $Q_i \in \mathbb{C}$ is the predicted coefficient corresponding to the i-th basis function $\boldsymbol{F}_i \in \mathbb{C}^D$, where $D = 3$ for near-fields and $D = 2$ for far-fields. The index i can always be truncated at some finite value, determined exclusively by the antenna's electrical size, without loss of field power. This approach offers several benefits over directly predicting EM fields:

- Physics compliance: As the NN model predicts SWE coefficients, its predictions are guaranteed to be valid solutions to Maxwell's equations.
- Compact representation: The SWE representation is often much more compact than one adequately sampling the EM field. This means that the output layer of the neural network can be similarly smaller. In practice, this can mean a 5-10x reduction in trainable parameters for small networks like the ones used in this work.
- Independent of evaluation point: Since only the basis functions depend on (r,θ,φ), it is possible to evaluate the field anywhere given only a single prediction of a set of SWE coefficients. The only caveat is that the evaluation point must lie outside the minimum sphere fully encompassing the antenna (see [3] for more details). Likewise, one can obtain both the near- and far-fields with a single prediction, as this simply requires choosing the appropriate basis functions. Besides being convenient at inference time, this greatly reduces the complexity of generating a dataset to train the NN model on, as we need not worry about adequate sampling of the sphere, let alone r moving from near- to far-field regions.

We have implemented spherical wave expansion in PyTorch [4] with caching of basis functions, allowing us to utilize automatic differentiation and GPU acceleration. This enables end-to-end model training and direct evaluation of both near- and far-fields from the model outputs.

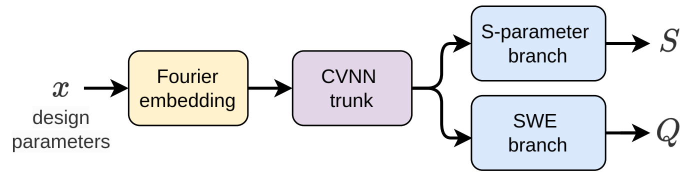

Fig. 1: Schematic of the neural network architecture used in this paper.

B. Tailored Neural Network Architecture

Our neural network takes as input a vector of real-valued design parameters, x, which together parameterize an antenna design. These can include both geometrical and electrical variables. From these, the network predicts:

- SWE coefficients, Q: These compactly represent an antenna's EM field while maintaining physical consistency with Maxwell's equations and enabling efficient near- and far-field reconstruction.
- S-parameters, S: The scattering parameters that characterize antenna performance metrics like return loss, transmission, and impedance matching.

A schematic of the network architecture can be seen in Fig. 1. A key challenge of its design was how to go from the real-valued design parameters to the complex-valued S and Q predictions. The details of the architecture will be discussed at the conference, but the main concepts can be found in [5], [6].

For training and evaluation of the network's performance, the predicted S-parameters and SWE coefficients are compared to ground truth values using the relative root mean squared error function (rRMSE) and combined in a multi-objective optimization scheme to balance performance between the two tasks. This is done using the AdamW optimizer [7] and a stepped exponentially decaying learning rate scheduler.

III. Case Study: Surrogate Modeling for a Slot Patch Antenna

To demonstrate the potential of the proposed NN-powered framework, the following case study considers the modeling of a complex slot patch antenna from [8]. The goal is to predict the SWE coefficients and the S-parameter S_{11} with high accuracy, enabling efficient and real-time performance evaluations for early-stage design exploration.

A. Antenna

The case study focuses on a two-layer parasitic patch antenna designed for Ka-band operation. The antenna consists of a driven patch, a parasitic patch, and two dielectric layers. The driven patch is square with chamfered corners and a tilted slot, achieving circular polarization with a single feed point. Similarly, the parasitic patch is square with a cross-shaped slot in the center, enhancing bandwidth and maintaining low cross-polarization discrimination over a broad frequency band centered around the operating frequency of 27.5 GHz. An example of the antenna design is shown in Fig. 2. The design parameters are listed in Table I.

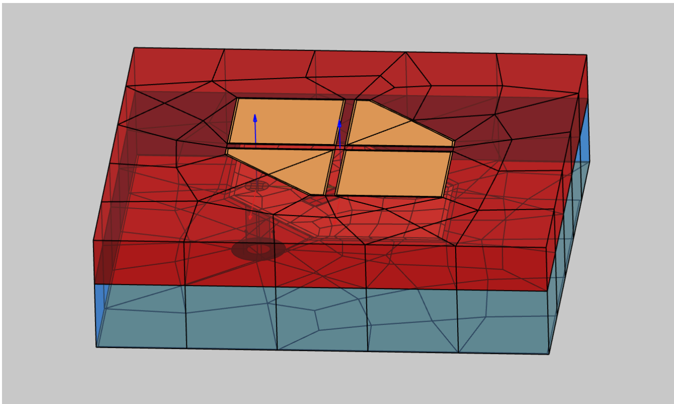

Fig. 2: Example of patch antenna used in the case study in TICRA Tools using ESTEAM [9].

B. Data Creation and Model Training

To train the NN-based surrogate model, a dataset of 15,000 samples was generated using a Latin hypercube sampling strategy. Each sample represents a unique antenna configuration, varying across key design parameters such as slot dimensions, patch scaling, and frequency. The data generation required approximately four days on a desktop with an AMD Ryzen 9 9950X CPU; the simulations were performed in TICRA Tools using ESTEAM [9] with an average CPU time of 23s per full-wave simulation. Table I summarizes the varied parameters and the bounds of the design space. A dataset size of 15,000 samples was chosen to balance model accuracy with the need to minimize computational effort, providing sufficient coverage of the parameter space without unnecessary data generation. The dataset was split into 70% training, 20% validation, and 10% test sets. The neural network surrogate uses the complex-valued architecture outlined in Section II. The model is trained to predict the first 70 SWE coefficients, sufficient to capture all field power, and the corresponding S-parameter S_{11}. To optimize performance, a hyper-parameter sweep was performed. The final network is a 1.2M parameter, 4.8MB CVNN with a total training time of around 5 minutes on an NVIDIA RTX 4090. The network is able to make microsecond predictions, making real-time design interaction possible, even when considering many-point frequency or parameter sweeps. In the broader context of deep learning, this network is fast, memory-efficient, and low-cost to train.

C. Results

The model's performance on the test set, consisting of samples the model has never seen, is shown in Table II. The predictive accuracy was assessed for subsets of the test set binned according to the reflection coefficient $|S_{11}|$ in dB. SWE predictions consistently achieve sub-0.5% rRMSE across all subsets. S-parameter prediction also reaches sub-1% errors overall but increases as the cut-off dB value decreases. This last observation could be explained by the fact that lower S-parameter values are rarer and will not have influenced the model as much during training. Still, when looking at the mean average error (MAE) of $|S_{11}|$ in dB, we see that this only translates to an error of 0.32 dB in the worst case.

TABLE I: Patch antenna surrogate parameter space

Parameter	Min	Max	Explanation
f_{ghz}	20.0	40.0	Driving frequency in GHz
L_{cut}	0.8	1.1	Normalized length parameter of patch
$L_{\text{cut,parasitic}}$	0.8	1.1	Normalized length parameter of parasitic patch
$L_{\text{patch,scale}}$	1.0	1.2	Scaling factor for patch
$\frac{L_{\text{patch}}}{L_{\text{parasitic}}}$	1.0	1.1	Ratio between size of patch and parasitic patch
W_{slot}	0.1	0.3	Width of gap in fed patch
$W_{\text{slot,parasitic}}$	0.05	0.15	Width of gaps in parasitic patch

To put these results into perspective, we fitted a multivariate RBF interpolator [10] on the training set as a baseline reference. This achieved errors of 5.4% and 6.9% rRMSE for SWE and S-parameter prediction, respectively. This underlines the potential of the proposed framework to achieve significantly higher accuracies than traditional methods given identical amounts of data.

Fig. 3 demonstrates the high level of agreement between predicted and simulated S-parameters, here as a sweep over frequency while keeping the other design parameters constant. The distribution of S-parameter values across the entire test set can be seen in Fig. 4. Finally, Fig. 5 shows an example near-field derived from predicted SWE coefficients and compares it to the true field, demonstrating the high accuracy of the surrogate model.

Overall, the model shows strong performance, especially given 1) the complexity of the antenna, which was chosen to represent a realistic setting, and 2) the wide frequency band on which the model was trained. This shows significant promise for utilizing this technology to enable real-time antenna design.

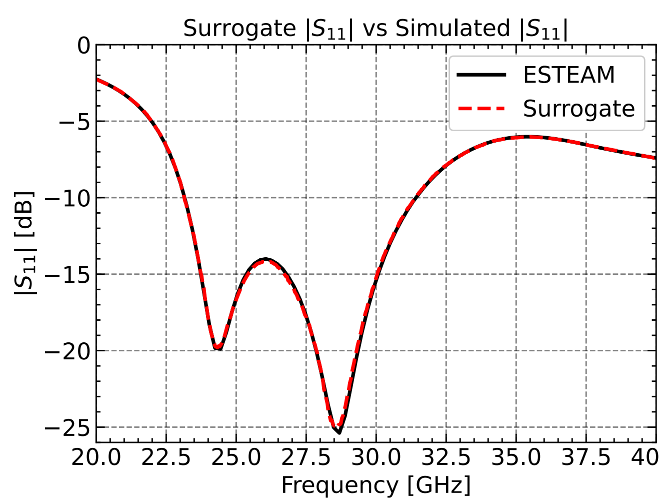

Fig. 3: Example $|S_{11}|$ frequency sweep, comparing surrogate and simulation (ESTEAM [9]) results.

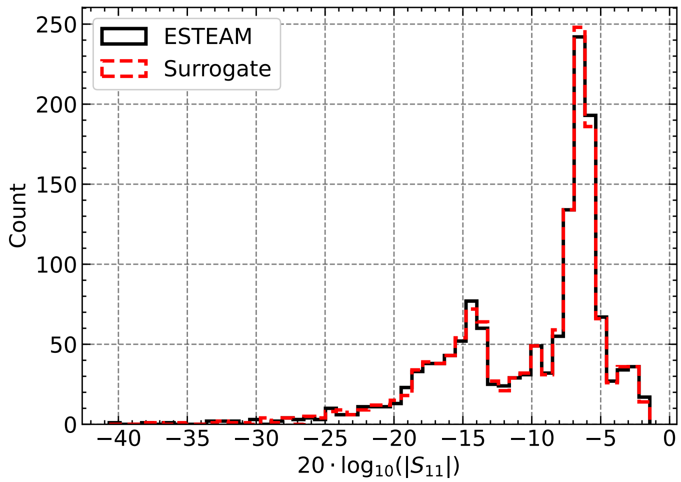

Fig. 4: Simulation (ESTEAM [9]) and surrogate $|S_{11}|$ test set distributions.

TABLE II: Model performance on the test dataset

Test subset	Subset size	SWE rRMSE	$\lvert S_{11}\rvert$ rRMSE	$\lvert S_{11}\rvert$ MAE
$\lvert S_{11}\rvert \leq 0$ dB	1455 (all)	0.49%	0.72%	0.06 dB
$\lvert S_{11}\rvert \leq -5$ dB	1315	0.47%	0.76%	0.07 dB
$\lvert S_{11}\rvert \leq -10$ dB	573	0.39%	1.73%	0.12 dB
$\lvert S_{11}\rvert \leq -15$ dB	308	0.39%	2.61%	0.16 dB
$\lvert S_{11}\rvert \leq -20$ dB	88	0.34%	5.57%	0.32 dB

IV. Conclusion

The proposed NN-powered framework offers an accurate and fast alternative to traditional methods for high-dimensional antenna design and exploration. By simultaneously predicting SWE coefficients and S-parameters, the framework produces compact, physics-compliant representations of electromagnetic fields, enabling real-time performance evaluations. Additionally, the integration of specialized complex-valued neural networks enhances its ability to handle phase-dependent data, a critical requirement for precise EM modeling. The framework demonstrates high numerical accuracy on a non-trivial multilayer patch antenna case study, achieving overall relative root mean square errors below 1% compared to simulation data for both SWE and S-parameter prediction.

By enabling accurate and real-time feedback on design parameter changes, the proposed framework shows significant promise for accelerating the iterative design process. This would allow engineers to efficiently explore complex parameter spaces, shorten design cycles, and tackle modern antenna design challenges.

Future work will explore integrating gradient-based optimization workflows and using active learning strategies to enhance robustness and training data efficiency further. Additionally, extending the framework to accommodate multi-antenna configurations and broader parameter ranges could unlock new capabilities for early-stage design and system-level optimization.

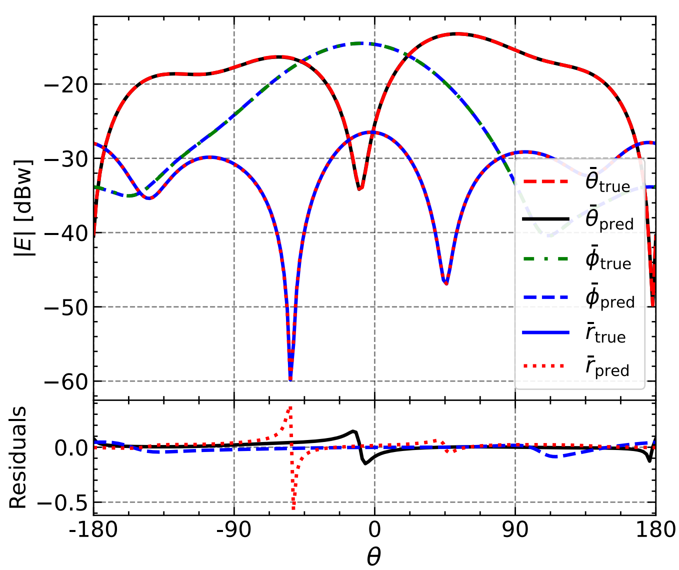

Fig. 5: Example of near-field reconstructed from predicted SWE coefficients and the true field. The error for this particular field is rRMSE = 0.3%.

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