Higher-Order Array Decomposition Method for Array Antennas with Connected Elements

Magnus Brandt-Møller[(1)(3)], Michael Mattes[(1)], Olav Breinbjerg[(2)], Min Zhou[(3)], and Erik Jørgensen[(3)]

[(1)] EMS, Technical University of Denmark, Kgs. Lyngby, Denmark, {magbran, mmattes}@elektro.dtu.dk
[(2)] ElMaReCo, Copenhagen, Denmark, olavbreinbjerg@outlook.com
[(3)] TICRA, Copenhagen, Denmark, {mbm, mz, ej}@ticra.com

***Abstract*—The Higher-Order Array Decomposition Method (HO-ADM) is extended to handle regular array antennas with interconnected elements. The Discontinuous Galerkin Method (DGM) is used to retain the multi-level block-Toeplitz Method of Moments (MoM) matrix structure, even for connected elements. The presented method yields more than an order of magnitude reduced solution time for a 8×8 real-world antenna array and similar memory consumption compared to existing fast methods.**

I. Introduction

Future array antennas will include a huge number of densely packed elements, resulting in electrically massive structures with strong edge effects and mutual coupling occurring. To accurately take into account these effects for electrically large arrays, traditional methods such as the embedded element pattern approach no longer suffice.

When analyzing arrays by solving surface integral equations (SIE) using the full-wave Method of Moments (MoM), all mutual coupling and edge effects are taken into account. This is nevertheless at the expense of a computational complexity and memory consumption which scales as $\mathcal{O}(N^2)$ and $\mathcal{O}(N^2) - \mathcal{O}(N^3)$, respectively, where N is the number of unknowns. By means of error-controllable approximations, various effective full-wave analysis techniques have been proposed, in which memory consumption and computational complexity can be reduced to as low as $\mathcal{O}(N \log N)$.

Widely used examples include the Multi-Level Fast Multipole Method (MLFMM) [1], the Adaptive Integral Method (AIM) [2], and the pre-corrected fast Fourier transform (pFFT) [3], the latter two require projection of BFs on a regular grid. For Macro-Basis Function (MBF) based methods [4], [5], the generation and number of MBFs to include is problem specific and the error cannot be controlled a priori.

When array elements are placed on a regular lattice the computation efficiency can be improved without compromising accuracy by employing the Array Decomposition Method (ADM) [6]. Recently, the boundary integral part of ADM has been implemented with higher-order (HO) basis functions and shown to use significantly less unknowns for a given accuracy [7]. Nevertheless, a restriction in the HO-ADM has been that no conduction current was allowed between elements, i.e. that elements could not be connected, effectively excluding arrays with a ground-plane or other interconnecting features. In this paper the existing HO-ADM has been extended, enabling it to handle electrically connected arrays.

II. HO-ADM with Connected Elements

The HO-ADM makes use of consecutively ordered HO basis functions (BFs), the regular arrangement of array elements, as well as the translational invariance of the 3D free-space Green function, to enable an FFT-accelerated matrix-vector product (MVP) used in the iterative solution procedure. This acceleration is only possible due to the resulting multi-level block-Toeplitz (MBT) MoM matrix $\mathbf{A}$. In case of electrically connected array elements this Toeplitz property is lost because the BF coefficients on connecting edges have to be associated with either one or the other array element. In the present contribution, the Discontinous Galerkin Method (DGM) [8] for surface integral equations is employed to keep the MBT property of $\mathbf{A}$. Consequently, the MVP can be FFT-accelerated in the case of connected array elements.

More specifically, the DGM is employed to split the roof-top BFs into half roof-tops at edges associated with two mesh-cells which belong to two different array elements. As a consequence, twice the number of BF coefficients are needed at connecting edges, but they can now be distributed evenly between array element interaction-matrix blocks in $\mathbf{A}$. To make $\mathbf{A}$ fully block-Toeplitz, however, a number of dummy BF need to be added to the array unit cell. These are half roof-tops but are placed on all external edges, i.e. edges which are only associated with a single mesh-cell. By doing so, self-interaction blocks for all array elements become equal, which in turn means that the MBT property of $\mathbf{A}$ is retained.

It should be noted that placing additional dummy unknowns at external edges changes the Krylov subspace and therefore also the obtained solution. However, by hiding the dummy BFs from the iterative solver, which is done by zeroing appropriate entries in the residual vector calculation, the solution is unchanged. As such, dummy unknowns are never solved for and merely serve to preserve the MBT property of $\mathbf{A}$ in order to accelerate the MVP using the FFT.

In summary, by splitting roof-tops into half roof-tops using the DGM to enforce current continuity and by introducing dummy unknowns, which are hidden from the iterative solver, electrical conduction currents are now allowed to flow between array elements in the extended HO-ADM.

III. Application Example

As a validation case, we consider an 8×8-element dual-frequency right-hand circularly polarized (RHCP) high-gain antenna sub-array based on the design in [9] (Fig. 1) with

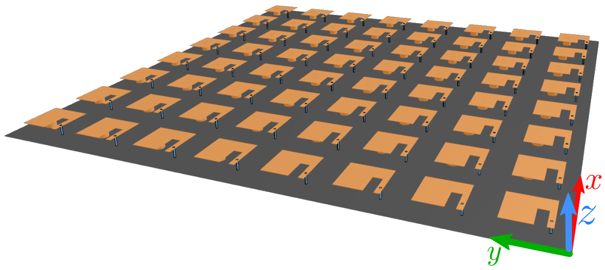

Fig. 1. 8×8-element (area of $28.5\lambda^2$) dual-frequency right-hand circularly polarized (RHCP) high-gain antenna sub-array from [9], with radiation pattern coordinate system.

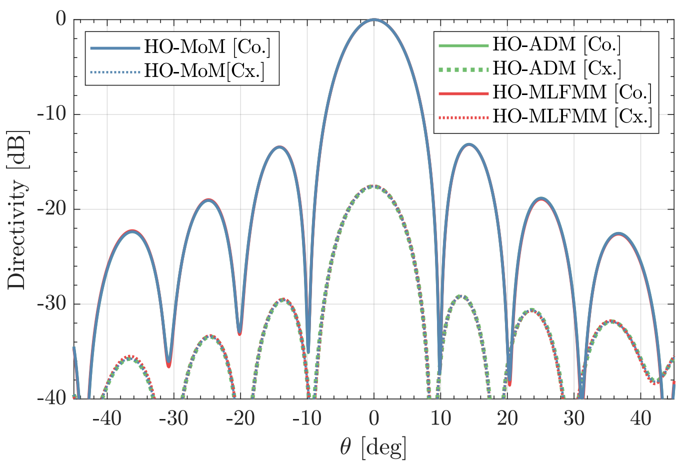

Fig. 2. Normalized far-field directivity pattern ($\phi = 0°$-cut) for the sub-array in Fig. 1 at 8.425 GHz, comparing the extended HO-ADM to HO-MLFMM and HO-MoM. Co-polarization is RHCP whereas the cross-polarization is LHCP.

64 independent wire excitations. The sub-array is meshed with a total of 7,552 quadrilaterals comprising $N = 60{,}832$ unknowns in the HO-MLFMM/HO-MoM, while $N = 62{,}464$ unknowns are needed in the HO-ADM due to the DGM and dummy unknowns as described in Section II.

The radiated far-field pattern in Fig. 2 has been computed at 8.425 GHz on a laptop with an Intel® Core® i7-9850H CPU @ 2.6 GHz with 6 cores. Comparisons are made with the full-wave solver in ESTEAM [10] which is based on a state-of-the-art HO-MoM/MLFMM implementation. The calculated peak directivity for all methods is 26.07 dB, which is close to the reported measured directivity of 26 dB [9]. In Fig. 2, both the co- and cross-polarization $\phi = 0°$-patterns are seen to coincide comparing the HO-ADM with HO-MoM, effectively verifying the outlined procedure in Section II. There is equally good agreement in other ϕ-cuts.

Table I shows a comparison of solution time and memory consumption between the HO-ADM and ESTEAM. Herein, the total solution time refers to generation of BFs, matrix filling, preconditioner generation and the iterative solution time. Total memory consumption refers to storage of required matrices, the Krylov subspace as well as the preconditioner.

For reference, the HO-MoM solution has been included

TABLE I
COMPARISON OF SOLUTION TIME AND MEMORY CONSUMPTION ON AN INTEL® CORE® I7-9850H CPU @ 2.6 GHz WITH 6 CORES.

Method	Total Simulation Time	Memory Consumption	Number of Iterations
HO-MoM	1 h 12 min	14 GB	N/A
HO-MLFMM	24 min	1.2 GB	245
Extended HO-ADM	41 s	1.6 GB	181

which takes 1 hour and 12 min to complete, with a memory consumption of 14 GB. By employing the more appropriate HO-MLFMM the solution time is around 24 min with a memory consumption of only 1.2 GB. At the penalty of slightly increased memory consumption (1.6 GB) using the extended HO-ADM, the solution time is reduced to 41 s.

IV. CONCLUSION

We presented an extended Higher-Order Array Decomposition Method capable of analyzing connected antenna arrays and applied it to a 8×8 sub-array designed for the Europa Lander mission. The results show a substantial computational speed-up by a factor of 35 compared to HO-MLFMM. This speed-up is achieved while maintaining a memory consumption comparable to HO-MFLMM. Results for the full 32×32-element array will be presented at the conference.

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