Efficient Full-Wave Computation of the Monostatic Radar Cross Section

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***Abstract*—A full-wave solver for computing monostatic radar cross sections of electrically large structures is proposed. The solver is based on method of moments. It uses the multilevel fast multipole method in combination with higher-order basis functions, a blocked iterative solver, and a compression of the right hand side matrix, in order to reduce the computational demand. The accuracy and efficiency of the method are demonstrated on two benchmark examples.**

I. Introduction

Computing the monostatic Radar Cross Section (RCS) is an important part of the engineering process in many different applications. For electrically large structures asymptotic methods are commonly used [1]. However, in many applications solving the full-wave problem is required to achieve the desired accuracy. This is a challenge for electrically large structures and has until recently been considered too computationally demanding.

The usage of expensive hardware has been considered in order to mitigate the high computational demand [2]. Expensive hardware is required when using integral equation based methods, as for example the Method of Moments (MoM), since it has the computational complexity $\mathcal{O}(N^3)$, where $N = \mathcal{O}(f^2)$ is the number of unknowns and f is the frequency.

To avoid this use of expensive hardware, one can reduce the computational complexity of full-wave methods by using acceleration techniques, such as the Multi-Level Fast Multipole Method (MLFMM). In combination with higher-order basis functions, this can significantly reduce the computational demand [3]. The challenge when using acceleration techniques for RCS computations is that one needs to rely on an iterative solver to obtain the solution. However when considering RCS one needs to solve the system with a different right hand side for each incident angle. Hence, the solution time scales with the number of incident angles P. The complexity of solving the system when using MLFMM is $\mathcal{O}(PN_{it}N\log N)$, where N_{it} is the number of iterations of the iterative solver.

This work builds on the method in [4] and we propose a numerical method with several measures to reduce the computational effort. 1) We use higher-order basis functions to reduce the number of unknowns N for a given accuracy. 2) We do a compression of the right hand side matrix to reduce the number of right hand sides P. 3) We use a blocked iterative solver to reduce the number of iterations per right hand side N_{it}. In combination these techniques allow for efficient and accurate full-wave computation of the RCS for electrically large structures.

We demonstrate the accuracy of the method by computing the RCS of a sphere where an analytical solution is known and demonstrate the efficiency by computing the RCS of the NASA almond at 100 GHz using standard hardware.

II. Numerical Method

Taking into account the polarizations of the incident and scattered fields, the RCS in a given direction (θ_u, ϕ_u) can be defined as

$$\sigma(\theta_u, \phi_u)_{\psi\nu} = \lim_{r\to\infty} 4\pi r^2 \frac{|\boldsymbol{E}^S(\theta_u, \phi_u) \cdot \hat{\boldsymbol{\psi}}|^2}{|\boldsymbol{E}^I(\theta_i, \phi_i) \cdot \hat{\boldsymbol{\nu}}|^2}. \tag{1}$$

Here, $\boldsymbol{E}^I(\theta_i, \phi_i) = \boldsymbol{E}_0 e^{-jk_0 \hat{\boldsymbol{k}}\cdot\bar{r}}$ is the incident electric field due to a plane wave from the direction (θ_i, ϕ_i) with the plane-wave amplitude $\boldsymbol{E}_0$, propagation vector $\hat{\boldsymbol{k}} = -(\sin\theta_i \cos\phi_i \hat{\boldsymbol{x}} + \sin\theta_i \sin\phi_i \hat{\boldsymbol{y}} + \cos\theta_i \hat{\boldsymbol{z}})$, and free-space wavenumber k_0. $\boldsymbol{E}^S(\theta_u, \phi_u)$ is the scattered far field in direction (θ_u, ϕ_u). The polarization vectors are chosen as the spherical unit vectors $\hat{\boldsymbol{\psi}}, \hat{\boldsymbol{\nu}} = \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}$. For the rest of the paper we consider the monostatic case $(\theta_u, \phi_u) = (\theta_i, \phi_i)$.

A. Integral Equations and Their Discretization

The full-wave problem can be solved using the integral equations for time harmonic fields. In particular we consider a perfectly conducting scatterer $\mathcal{S}$ and use the mixed potential Electrical Field Integral Equations (EFIE). For closed parts of $\mathcal{S}$ we apply the Combined Field Integral Equations (CFIE).

Using a Galerkin approach for the discretization we arrive at the linear system

$$\overline{\overline{Z}}\,\overline{\overline{I}} = \overline{\overline{V}}, \tag{2}$$

where the impedance matrix $\overline{\overline{Z}}$ is $N \times N$. The solution matrix $\overline{\overline{I}}$ and the right hand side matrix $\overline{\overline{V}}$ have size $N \times P$, where P is the number of incident angles.

We employ higher order basis functions for the discretization to reduce the number of unknowns N significantly for a given accuracy, compared to standard RWG basis functions [3].

B. Solution Method

To accelerate matrix vector products with the impedance matrix $\overline{\overline{Z}}$, we use MLFMM. This means that we need to employ an iterative solver when solving (2).

We use a blocked GMRES as the iterative solver [5]. The advantage of the blocked version is that the Krylov space is shared across all right hand sides, which results in a fewer number of iterations per right hand side.

To reduce the number of right hand sides we apply a compression to the right hand side matrix

$$\overline{\overline{V}} \approx \overline{\overline{C}}\,\overline{\overline{D}} \tag{3}$$

TABLE I

RESULTS FOR THE FULL-WAVE RCS COMPUTATION OF THE TWO CONFIGURATIONS. $\rho_{\max}$ THE RADIUS OF THE MINIMUM ENCLOSING SPHERE, N THE NUMBER OF UNKNOWNS, P THE NUMBER OF RIGHT HAND SIDES AND P_c THE NUMBER OF RIGHT HAND SIDES AFTER COMPRESSION.

Case	order	$\rho_{\max}$	N	P	P_c	time	RMS
Sphere	5	3.3λ	4296	84	59	18.6 sec	0.055
Sphere	6	3.3λ	6312	84	59	49.1 sec	0.008
Sphere	7	3.3λ	8712	84	65	2:04 min	0.0009
Almond	6	41.7λ	171921	1080	365	3:06 hrs	-

where $\overline{\overline{C}}$ has size $N \times P_c$ and $\overline{\overline{D}}$ has size $P_c \times P$, and $P_c < P$ is the compressed number of right hand sides. This allows us to only apply the iterative solver to P_c right hand sides, while still being able to compute an accurate RCS for all incident angles.

The final complexity of the method is $\mathcal{O}(P_c N_{it} N \log N)$.

III. Numerical Examples

All the results in this section are generated on a laptop from 2020, with an i7 12 core 2.7 GHz CPU and 32 GB of memory.

A. Sphere

In this section we consider a sphere of radius 1 m at 1 GHz. The computed monostatic RCS for different polynomial orders is shown in Fig. 1. We see that when the polynomial order is increased the computed RCS converges to the analytical solution.

As seen from Table I the RCS can be computed efficiently due to the low number of unknowns, and due to the reduced number of right hand sides after the compression.

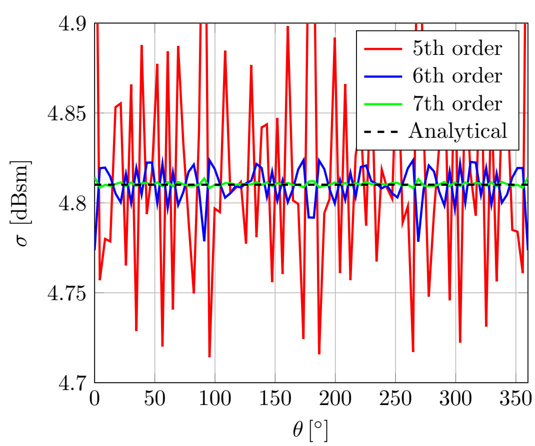

Fig. 1. Monostatic RCS for the sphere discussed in Section III-A for different polynomial orders of the basis functions. The dashed line shows the analytical result. Note the narrow range of the y-axis.

B. Electrically Large NASA Almond

Finally, we consider the classic NASA Almond for the frequency of 100 GHz, where the electrical size is 41.7λ. For this electrically large problem we are still able to compute the monostatic RCS in around 3 hours on a standard laptop, as seen in Table I. The computed RCS for a $\hat{\phi}$-polarization incident and scattered field is plotted in Fig. 2 and the mesh is shown in Fig. 3.

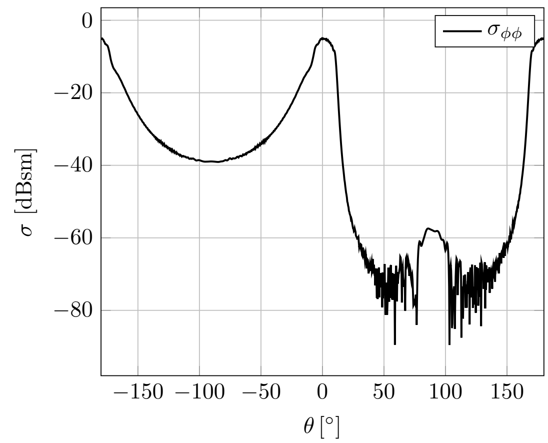

Fig. 2. Monostatic RCS for the NASA almond discussed in Section III-B, at 100 GHz.

IV. Conclusion

A full-wave solver for computing the monostatic RCS of electrically large structures was proposed. The method combines MLFMM with higher order basis functions, a blocked iterative solver and compression of right hand sides to reduce the computational demand. The accuracy and efficiency were demonstrated on two benchmark problems and more examples will be given at the presentation.

References

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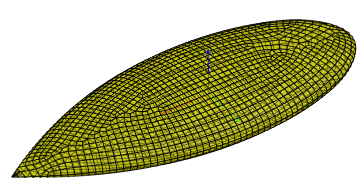

Fig. 3. The MoM mesh for the almond discussed in Section III-B. The red arrow indicates the x-direction, the green the y-direction and the blue the z-direction.