Performance of Array Decomposition Method with Higher-Order Basis Functions

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***Abstract*—The performance of the Array Decomposition Method for finite regular antenna arrays with arbitrary identical elements using higher-order basis functions is investigated. We demonstrate how using higher-order basis functions results in significantly reduced simulation time for a $\mathbf{10 \times 10\,(22\lambda \times 22\lambda)}$ circular horn array, without the need for approximations.**

I. Introduction

Traditionally, antenna arrays have been employed in radar applications, radio astronomy and as feeds for reflector based systems. For such applications, traditional design approaches based on embedded element patterns and array factors or variants thereof have sufficed. With the on-going move from large spacecrafts in the geostationary orbit (GEO) to smaller spacecrafts in low earth orbits (LEO), as well as an increasing demand for flexible in-orbit configurations, direct radiating arrays (DRA) are employed more frequently for space missions. DRA commonly comprise densely packed elements which in turn demands more accurate modelling of edge and mutual coupling effects. In addition, stringent performance requirements in space substantiates the necessity of rigorous full-wave numerical methods.

Conventional full-wave methods, e.g. the Method of Moments (MoM), suffer from excessive memory requirement and computational complexity, $\mathcal{O}(N^2)$ and $\mathcal{O}(N^2) - \mathcal{O}(N^3)$ respectively, where N is the number of unknowns.

Several iterative methods have been proposed for the solution of electrically large arrays, in which the memory and computational requirement is $\mathcal{O}(N \log N)$. Examples include the Multi-Level Fast Multipole Method (MLFMM) [1], the Adaptive Integral Method (AIM) [2], and the pre-corrected fast Fourier transform (pFFT) [3]. While these methods accelerate the analysis of general arrays, it has been shown that exploiting the geometry of regular antenna arrays significantly reduce both memory requirement and computation time [4].

The present work concerns the MoM solution of electrically large antennas arrays with arbitrarily shaped, regularly spaced, identical and perfectly electrically conducting identically oriented elements, a common configuration for modern antenna arrays. The translational invariance of associated Green's function, together with the regular geometrical structure of the array, result in a block-Toeplitz matrix [5] which allows for a Fast Fourier Transform (FFT)-accelerated matrix-vector product (MVP) [4], [6] in the iterative solution, using the Array Decomposition Method (ADM).

ADM scales as the square of the number of basis functions per array element [6]. It is therefore paramount to keep the number of basis functions as low as possible for a fixed solution accuracy. Several approaches to reduce the sensitivity of ADM to increasing number of basis functions have been proposed [4], [7]; however, they are based on approximations, which may impact solution accuracy.

We demonstrate how the number of basis functions per array element can be reduced using higher-order (HO) basis functions [8]. Consequently, used in combination with ADM, a significant reduction in computation time can be achieved. To demonstrate this, the HO-ADM is applied to a $10 \times 10\,(22\lambda \times 22\lambda)$ circular horn array, resulting in much lower computation times compared to the ADM using first-order basis functions, without the need for approximations.

II. The Array Decomposition Method

This section is a distilled version of ADM [4], for the purpose of understanding ADM's quadratic scaling in number of basis functions per array element. We take outset in an arbitrary $p \times q$ element planar array, and write the total number of unknowns as $N = ST$, where S is the total number of unknowns per array element, and $T = p \times q$ is the total number of array elements.

Since the MVP with a (circulantly extended) block-Toeplitz matrix is equivalent to a convolution operation on the blocks, the MVP can be accelerated by the FFT. However, the individual matrix blocks of size $S \times S$ do not, in general, possess any special symmetry, and can therefore not be accelerated by the FFT. Thus, the computational complexity of ADM scale as $\mathcal{O}(S^2 T \log(T))$ rather than $\mathcal{O}(N \log N)$.

Moreover, in order to perform the MVP, ADM needs to store $(2p-1) \times (2q-1)$ blocks of size $S \times S$, resulting in an asymptotic memory scaling of $\mathcal{O}(S^2 T)$. With this asymptotic scaling, it is critical to keep S as low as possible without impacting solution accuracy. This can be achieved using higher-order basis functions. In this work, an ADM implementation using the higher-order hierarchical Legendre basis functions from [8] is used.

III. Results

We consider a $22\lambda \times 22\lambda$ direct radiating array (Fig. 1) which consists of 10×10 circular horn antennas fed by circular waveguides excited with the fundamental TE_{11} mode. The radiated far-field pattern has been calculated using HO-ADM on

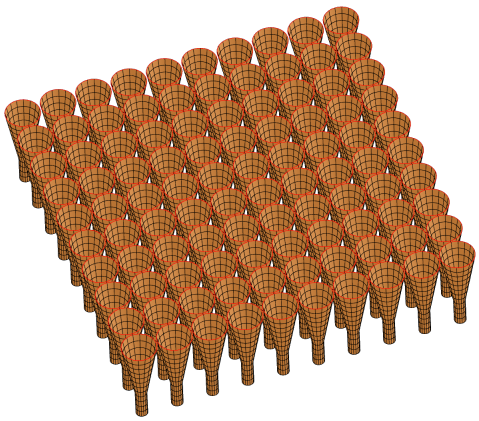


Fig. 1: 10×10 element circular horn array. 2.1λ aperture diameter. 5.6λ horn height. 2.2λ inter-element distance.

a computational machine with an Intel®Xeon®5218 CPU @ 2.3 GHz with 16 cores. A reference solution has been generated using the smallest mesh length possible on the available system. For fixed basis function order p, the maximal admissible mesh length has been varied between 0.15λ and 1.5λ to ensure an RMS error in the radiated far-field forward hemisphere that is less than $1\,\%$ (far-field requirement).

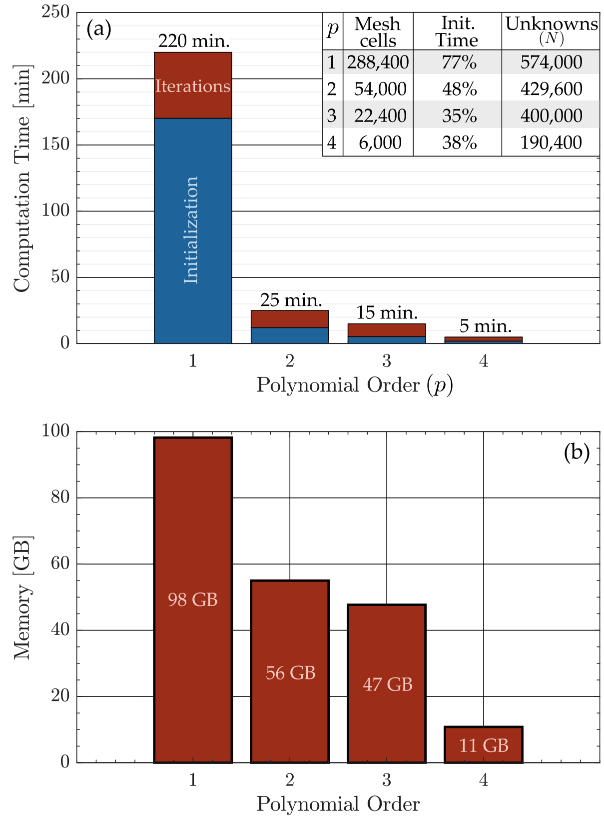
Fig. 2 (a) shows the total computation time (including initialization and iterative solution) and memory usage (b), for different fixed basis function orders p. For each order p, the maximal admissible mesh length has been decreased until reaching the far-field requirement (or lower). For $p = 1$, a total of 288,400 mesh cells ($\approx 0.15\lambda$) are needed to satisfy the far-field requirement, which is considerably more than the 54,000 mesh cells ($\approx 0.3\lambda$) needed for $p = 2$. The high number of mesh cells for $p = 1$ results in high ADM initialization time, primarily due to the increased number of integrals to compute.

The significant difference in computation time from 220 min. ($p = 1$) to 25 min. ($p = 2$) can be explained primarily by the decrease in the number of mesh cells. Notably, due to meshing constraints, the mesh is more refined for $p = 2$ and $p = 3$, resulting in a two and four times lower RMS error, respectively, than the solution for $p = 1$ and $p = 4$. The increased accuracy is the primary reason for the relatively small decrease in total number of unknowns, memory and computation time from $p = 2$ to $p = 3$.

Overall, the results clearly demonstrate superior performance when increasing the basis function order. This is most clearly seen in the transition from $p = 1$ and $p = 2$, where the total computation time decreases by a factor of 9, the memory decreases by a factor of two, even while the RMS error is halved.

IV. Conclusion

A reduction in required memory and computation time for simulation of regular arrays has been demonstrated. This is achieved by means of the Array Decomposition Method used in conjunction with higher-order basis functions. It is, to the best of the authors' knowledge, the first time such a combination has been implemented. The results show that using this approach, the memory and computation time savings in the order of 9 and 44 times, respectively, can be achieved compared to traditional first-order basis function implementations.



p	Mesh cells	Init. Time	Unknowns (N)
1	288,400	77%	574,000
2	54,000	48%	429,600
3	22,400	35%	400,000
4	6,000	38%	190,400

Fig. 2: Total computation time (a) and memory usage (b) for HO-ADM required to reach $< 1\,\%$ far-field RMS error.

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