

Random surface distortion in GRASP



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$$L_0 \mathbf{J}_s = j\omega\mu_0 \left(\int_S \mathbf{J}_s(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') dS' \right. \\ \left. + \frac{1}{k_0^2} \int_S \nabla'_s \cdot \mathbf{J}_s(\mathbf{r}') \nabla G_0(\mathbf{r}, \mathbf{r}') dS' \right),$$

$$\hat{\mathbf{n}} \times \mathbf{E}' = \hat{\mathbf{n}} \times L_0 \mathbf{J}_s, \quad \mathbf{r} \in S$$



Introduction

Surfaces with random distortions are encountered in many practical antenna problems, e.g., when a reflector antenna is manufactured to a specific surface tolerance or it is subjected to wear and tear during its lifetime. As increasingly higher frequencies are used in reflector systems, it becomes more and more important for the antenna engineer to be able to accurately estimate the effects of surface distortions.

In this white paper, we describe how the scattered field from a surface with random distortions can be calculated by GRASP and how these results, as shown in more detail in [1], correspond to the widely used Ruze equations [2].

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Random surface error model

Random surface errors in GRASP are defined by the two values ns and ϵ_p . The random surface is generated by covering the reflector by a regular grid with a node spacing ns along both axes of the grid. The surface offset at the nodes of the grid is selected as random numbers uniformly distributed in an interval with the peak-to-peak value $2\epsilon_p$ and a mean value of zero. A cubic interpolation function connects the grid points and yields a continuous surface between the random values at the nodes. This means that within a square with side lengths $2ns$ the surface distortions are correlated, whereas they are nearly uncorrelated for larger distances.

It can be shown that the cubic interpolation used in the GRASP random surface leads to an rms value, ϵ_{rms} equal to the selected peak value, ϵ_p , times a correction value stemming from the cubic interpolation of 0.47, i.e.,

$$\epsilon_{rms} = 0.47 \cdot \epsilon_p \quad (1)$$

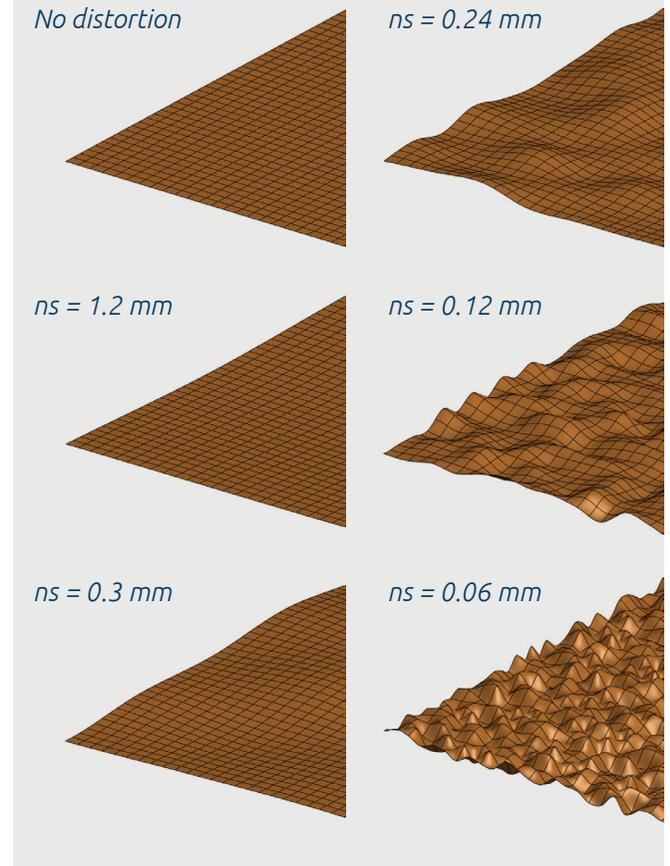
The spacing between the nodes, ns , determines the smoothness of the surface. A larger value of ns results in a slower variation across the surface, and thus a smoother surface. On the other hand, smaller values of ns provide a more rapidly varying surface.

Five cases of surface smoothness corresponding to $ns = 1.2, 0.6, 0.24, 0.12,$ and 0.06 mm for the same value of ϵ_{rms} are illustrated in **Figure 1**.

Correlation length

A common source of confusion when dealing with rough surfaces is the different definitions of correlation length used in different professions. In his examinations of reflector antennas, Ruze [2] based his equations on a concept of a correlation region with diameter "2c" outside of which the correlation is zero. Another more general definition of the correlation length L_c , which is widely used in optics, is the distance from the peak of the autocorrelation function, ACF, at which the ACF has dropped to a value e^{-1} below the peak [3].

Figure 1 – The surface shape for different smoothness. The distance between the black lines on the surface is 0.06 mm. Only a corner of the plate is shown



To illustrate the two different definitions and how they relate to the node spacing ns used in GRASP, a rough surface with a grid spacing of $ns = 5$ mm and $\epsilon_p = 0.5$ mm is shown in **Figure 2**. The autocorrelation function, ACF, of this rough surface is shown in **Figure 3**.

The autocorrelation function in the cuts through the peak along x and y are shown in **Figure 4** and **Figure 5**, respectively.

The peak value of the autocorrelation function, $ACF(0) = 0.0544519$, corresponds to an rms surface error, $\epsilon_{rms} = \sqrt{ACF(0)} = 0.2333494\text{mm} \approx 0.47\epsilon_p$, which is in good agreement with the expression in (1).

The correlation length L_c , defined as the distance at which the value of the autocorrelation function is e^{-1} of the peak value, is $L_c = 3.86$ mm $\approx 0.77 ns$.

It is seen that the distance between the first two minima of the autocorrelation function is approximately 10 mm, or $2c$, thus corresponding well to Ruze's definition of correlation distance. In other words, it is sensible to compare the results of Ruze for a correlation region of $2c$ to those of GRASP with a node spacing of ns .

Figure 2 – The surface shape for 5 mm grid. Unit in mm

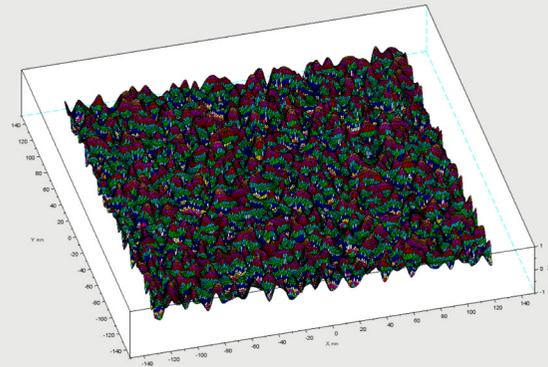


Figure 3 – Autocorrelation function for surface in Figure 2

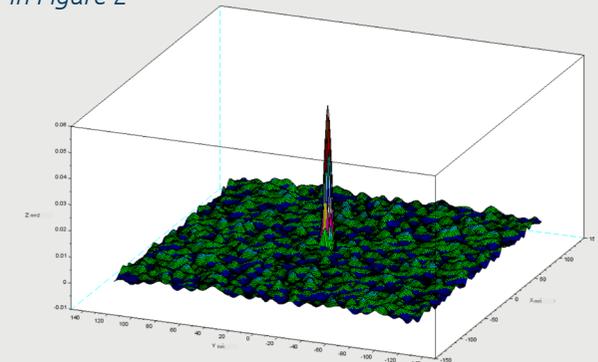


Figure 4 – Autocorrelation function for $y=0$ and the Gaussian approximation with $\sigma = 2.5$ mm

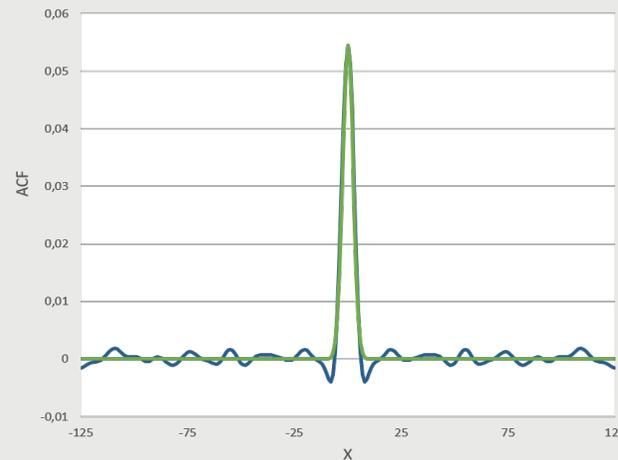
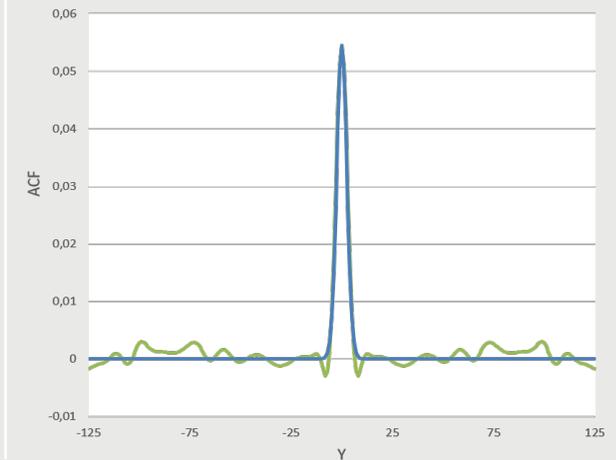


Figure 5 – Autocorrelation function for $x=0$ and the Gaussian approximation with $\sigma = 2.5$ mm



Ruze equation for RF performance

Random errors on a reflector surface will scatter the field from the forward direction into the side lobe region thereby reducing the peak gain and increasing the side lobes. The standard Ruze formula (2), which is derived for a plane wave incident on a planar surface with a random distortion with $2c \geq \lambda$, states that the peak gain is reduced by the factor

$$P_{planar} = e^{-\left(\frac{4\pi \epsilon_{rms}}{\lambda}\right)^2}, \text{ for } 2c \geq \lambda. \quad (2)$$

For a reflector antenna the equation must be changed to:

$$P_{aperture} = e^{-\left(\frac{2\pi \delta_{rms}}{\lambda}\right)^2}, \text{ for } 2c \geq \lambda. \quad (3)$$

where δ_{rms} is the root mean square aperture error in metric units and $2\pi \delta_{rms}/\lambda$ is the rms aperture field phase error, see **Figure 6**. Notice, that this gain reduction is in the boresight direction, normally $\theta = 0^\circ$.

For a simple offset reflector as in **Figure 6** the aperture error is found by:

$$\delta_{rms} = (1 + \cos(\nu)) \cdot \epsilon_{rms} \quad (4)$$

Figure 6 – Aperture error for offset reflector

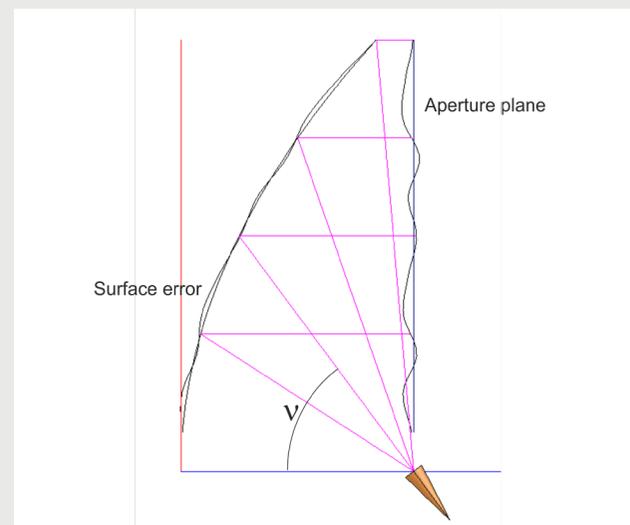
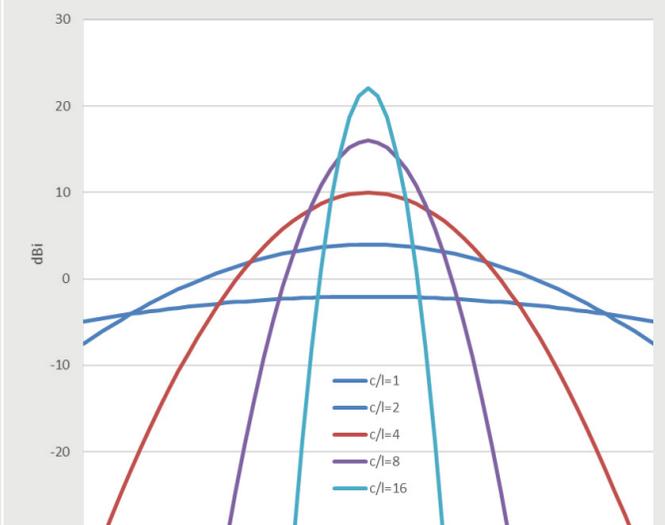


Figure 7 – Ruze distortion field for random surface error with $\delta_{rms} = 0.02\lambda$



The disturbance of the RF field outside the main beam is also estimated by Ruze, where the average level of the distortion field in the sidelobe region is given by

$$S_{refl} = \left(\frac{2\pi c}{\lambda}\right)^2 \cdot \left(\frac{2\pi \delta_{rms}}{\lambda}\right)^2 \cdot e^{-(\pi \cdot \sin(\theta))^2} \cdot (c/\lambda)^2, \text{ for } 2c \geq \lambda. \quad (5)$$

where θ is the angle measured from the boresight direction.

The relation is illustrated graphically in **Figure 7** for $\delta_{rms} = 0.02\lambda$. Note that the distortion pattern is independent of the reflector size. The expression is not valid near the axis, $\theta = 0^\circ$, where instead equation (3) should be used.

Gain-Loss Comparisons

In this chapter the gain loss is calculated by both Physical Optics (PO), which is available in GRASP, and the Method of Moments (MoM), which is available in ESTEAM, and compared with those predicted by the Ruze equations above. The analysed reflector is a plane PEC plate at the frequency of 500 GHz corresponding to a wavelength $\lambda = 0.6$ mm. The side length of the square PEC plate is 12 mm (20λ).

First, the plate is illuminated by a plane wave, which is the assumption for the Ruze equation, and secondly, by a tapered Gaussian beam.

Plane-Wave Illumination

The plate is illuminated by a plane wave orthogonally from the front as illustrated in **Figure 8**.

For reference, the reflected far field from the plate with no surface distortions is calculated by both Method of Moments (MoM) and Physical Optics (PO). The result is shown in the first line in **Table 1**. Both MoM and PO give a directivity of 38.06 dBi and the patterns obtained with MoM and PO are identical.

Peak loss as function of aperture error

Random errors on the reflector surface will scatter the field from the forward direction into the side lobe region thus reducing the peak gain and increasing the side lobes. The standard Ruze formula (2) states that the peak gain in the boresight direction is reduced by the factor

$$P_{\text{aperture}} = e^{-\left(\frac{2\pi \delta_{rms}}{\lambda}\right)^2}$$

where it is assumed that $2c \geq \lambda$. The limitation of this equation regarding the size of the surface error ϵ_{rms} is investigated first. A value of $c = 1.2$ mm and thus $2c = 4\lambda$ is selected and the surface roughness corresponding to the aperture errors of $\delta_{rms} = 0.063 \lambda$ to 0.251λ is used in the calculations.

Figure 8 – Distorted plate and plane-wave illumination

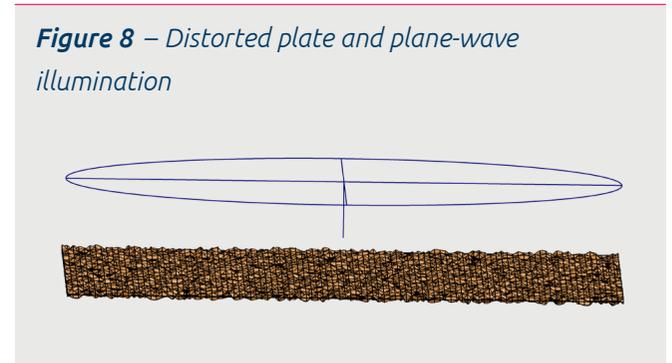


Table 1 – Peak directivity of the scattered beam as a function of the distortion peak.
Node spacing $ns = 1.2 \text{ mm} = 2\lambda$

Distortion Peak ϵ_p	Distortion rms ϵ_{rms}	Aperture error δ_{rms}	Ruze gain Loss dB	MoM solution		PO solution	
				Directivity dBi	Loss dB	Directivity dBi	Loss dB
mm	mm	λ	Loss dB	Directivity dBi	Loss dB	Directivity dBi	Loss dB
no dist.	no dist.	0	0	38.06	0.00	38.06	0.00
0.04	0.019	0.063	0.67	37.38	0.68	37.38	0.68
0.06	0.028	0.094	1.52	36.51	1.55	36.51	1.55
0.08	0.038	0.125	2.69	35.26	2.80	35.26	2.80
0.1	0.047	0.157	4.21	33.56	4.50	33.56	4.50
0.12	0.056	0.188	6.06	31.35	6.71	31.36	6.70
0.14	0.066	0.219	8.25	28.51	9.55	28.52	9.54
0.16	0.075	0.251	10.78	24.80	13.26	24.82	13.24

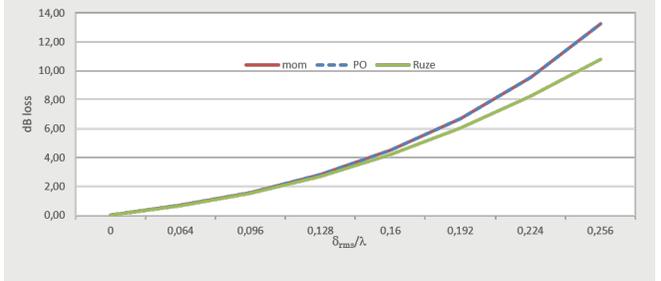
The scattered field is calculated by both MoM and PO and the resulting peak directivities are shown in columns 5 and 7 of **Table 1** for the MoM and the PO solutions, respectively. The loss relative to the non distorted surface is presented in columns 6 and 8. The gain losses as predicted by the Ruze equation in column 4 show that for a distortion larger than an aperture error of $\delta_{rms} = 0.1\lambda$ the estimates by Ruze are too low. This is due to the arising 2nd order effects in the basic Ruze equation. The agreement between MoM and PO is very good for all aperture errors, see **Figure 9**.

We conclude that if the aperture error $\delta_{rms} > 0.1\lambda$, Ruze's equation for gain loss is inaccurate.

Loss as function of node spacing ns

In the following we will investigate five cases of different node spacings $ns = 1.2, 0.6, 0.24, 0.12,$ and 0.06 mm as shown in column 1 of **Table 2**. A peak value of $\epsilon_p = 0.04 \text{ mm}$ giving a $\delta_{rms} = 0.032 \text{ mm}$ and a gain loss of 0.7 dB according to Ruze is selected. The resulting surface shapes are illustrated in **Figure 1**.

Figure 9 – Peak loss as function of aperture error.
Point distance $c = 1.2 \text{ mm} = 2\lambda$



It is clear that the assumption of a correlation distance larger than the wavelength, 0.6 mm , is not fulfilled for all correlation distances, as shown in column 2 of **Table 2**, and we can therefore not expect the Ruze formula to be valid for these correlation distances.

The scattered field is calculated by MoM and PO and the resulting peak directivities are shown in columns 4 and 6 of **Table 2** for the MoM and the PO solutions, respectively. The loss relative to the non distorted surface is presented in columns 5 and 7. The results

Table 2 – Peak directivity and loss of the scattered beam as a function of the node spacing ns .
Surface roughness $\epsilon_{rms} = 0.032 \lambda$

Node spacing ns	Correlation distance $2c$	Correlation length L_c	MoM solution		PO solution		Ruze gain
			Directivity dBi	Loss dB	Directivity dBi	Loss dB	
mm	λ	λ	Directivity dBi	Loss dB	Directivity dBi	Loss dB	Loss dB
no dist.	-	-	38.06	0.00	38.06	0.00	-
1.20	4.00	1.6	37.38	0.68	37.38	0.68	0.67
0.60	2.00	0.8	37.42	0.64	37.43	0.63	0.63
0.24	0.80	0.32	37.48	0.58	37.39	0.67	0.67
0.12	0.40	0.16	37.86	0.20	37.38	0.68	0.68
0.06	0.20	0.08	38.00	0.06	37.38	0.68	0.67

show that for node spacings of $ns = 1.2$ mm and 0.6 mm, where the diameter of the correlation region as defined by Ruze is larger than the wavelength, the agreement between MoM and PO is very good and the directivity loss is well predicted by Ruze. The difference in the results for $ns = 1.2$ mm and 0.6 mm is caused by the change of the number of sample points in the random surface distortion. To simulate a smaller correlation length, the number of random points must be increased.

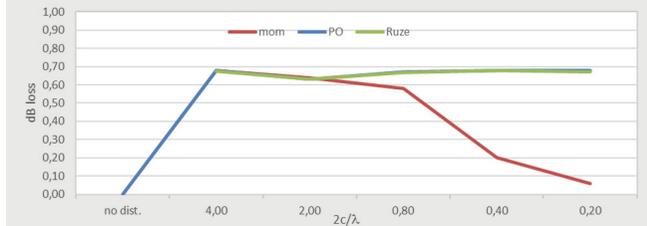
Decreasing the node spacing to $ns = 0.24$, 0.12, and 0.06 mm leads to deviating results in the MoM and PO solutions, and for the most rough surface, $c = 0.06$ mm,

the PO solution deviates significantly from the more correct MoM result as can be seen in **Figure 10**.

Another observation is that when the correlation distance becomes less than a wavelength, the loss in peak gain as predicted by the MoM analysis decreases and for $ns = 0.06$ mm it is only 0.06 dB. This behaviour can be explained by the following considerations.

If a plane wave illuminates a plane surface with periodic distortions with a period less than a wavelength, all the incident power will be reflected in the specular direction independent of the amplitude of the distortion. Imagine

Figure 10 – Peak loss as function of the correlation distance used by Ruze. Aperture error $\delta_{rms} = 0.064$ mm



now that we make a Fourier expansion of the surface in **Figure 8** for $ns = 0.06$ mm. This expansion will contain a number of harmonics with a period smaller than the wavelength which will therefore not contribute to the directivity loss. Only the harmonics with a period larger than the wavelength will contribute, and they will play a smaller role the faster the surface variations are. Therefore, the directivity loss decreases with decreasing c .

We conclude from the above that for $2c < \lambda$, both PO and the Ruze gain loss equation give too large values and MoM must be used to calculate the radiation pattern.

Tapered illumination

The influence of a tapered illumination, more similar to the feeds used in real-life reflector antennas, is investigated in this section. The flat plate from above is now illuminated by a Gaussian beam with six different waist sizes, w_0 , in the interval from 1 mm to 6 mm. The waists are located on the plate surface as illustrated in **Figure 11**.

This generates a tapered field on the plate with a constant phase, see **Figure 12**.

Figure 11 – Random distorted plate illuminated by a Gaussian beam. The waist is located at the nominal surface of the plate

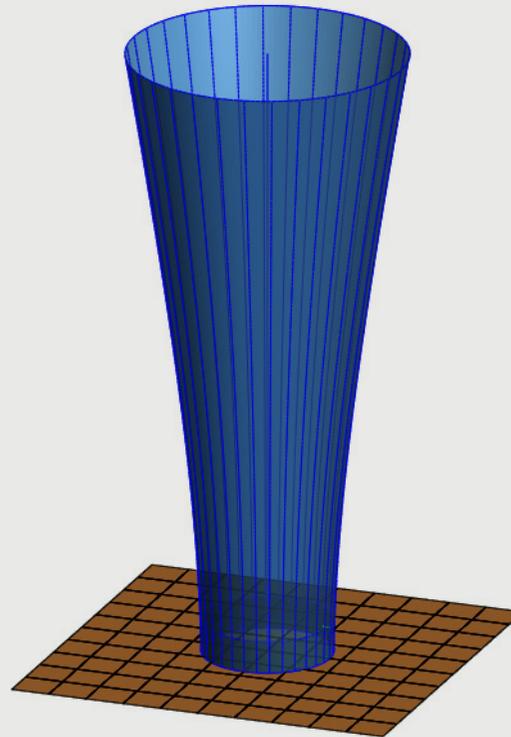


Figure 12 – Field strength (black) and phase (red) of Gaussian beam at the surface of the plate. Waist size $w_0 = 3\text{mm}$

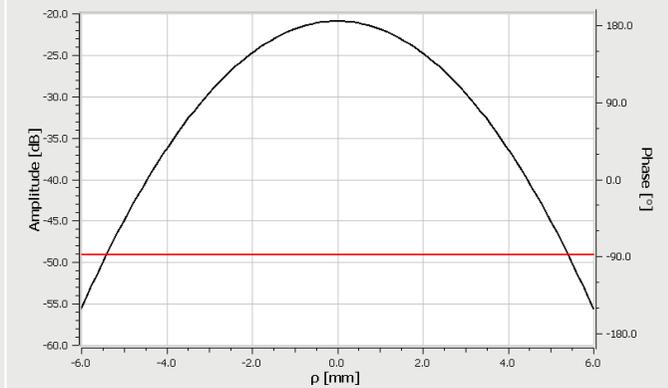


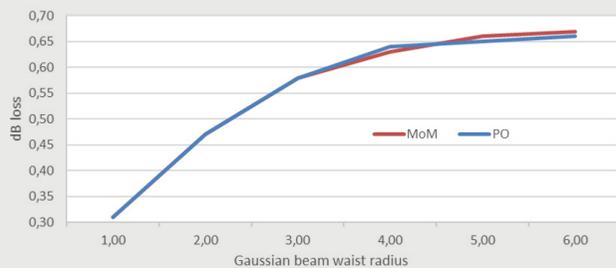
Table 3 – Gain loss of the scattered beam as a function of tapered illumination, $c=1.2$ mm.

Waist radius w_0	MoM solution	PO solution	Equivalent Ruze δ_{rms}
mm	Loss dB	Loss dB	λ
1.00	0.31	0.31	0.043
2.00	0.47	0.47	0.052
3.00	0.58	0.58	0.058
4.00	0.63	0.64	0.061
5.00	0.66	0.65	0.062
6.00	0.67	0.66	0.062

The random surface is created using $\epsilon_p = 0.04$ mm and a node spacing $ns = 1.2$ mm. The assumption of a Ruze correlation region with a diameter greater than the wavelength of 0.6 mm is fulfilled, and we would therefore expect the Ruze gain-loss equation to be valid. and the PO and MoM gain calculations to be identically, which is shown in **Table 3** and **Figure 13**.

In **Table 2** it was shown that Ruze's formulation, which does not account for the taper, would predict a loss of 0.67 dB for this case. We can derive an "equivalent" Ruze aperture rms error that would lead to the same gain loss as predicted by PO, and this is shown in column 4 of **Table 3**.

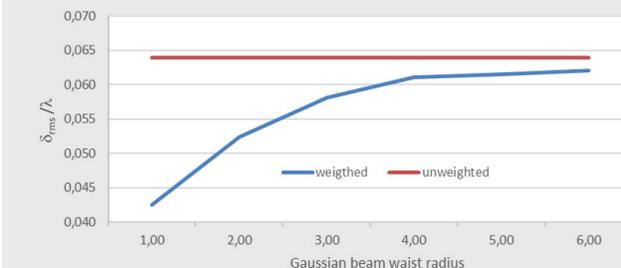
Figure 13 – Loss in peak calculated with PO and MoM as function waist radius w_0 . Node spacing $ns = 1.2$ mm



It is evident that the aperture rms error, δ_{rms} in the Ruze equation of the gain losses must be corrected with the weights given by a tapered illuminated field. The equivalent weighted and the unweighted aperture rms errors, δ_{rms} as a function of the Gaussian beam waists are shown in **Figure 14**.

When the Gaussian beam waist increases giving a constant illumination of the plate, the weighted aperture error is approaching the total rms value. The influence of the node spacing is considered by looking at the losses for five different node spacings, $ns = 1.2, 0.6, 0.24, 0.12,$ and 0.06 mm using a tapered

Figure 14 – Equivalent aperture error



illumination. In order to get a low illumination at the plate edges and still have a reasonable plate size a beam waist radius of 1 mm is selected.

First, we investigate the case where the plate has no surface distortions. The reflected peak gain from the plate calculated by both Method of Moments (MoM) and Physical Optics (PO) is shown in the first line in **Table 4**. Both the MoM and the PO solution give a directivity of 23.45 dBi.

Table 4 – Peak directivity of the scattered beam as a function of the node spacing, $\delta_{rms} = 0.064 \lambda$.

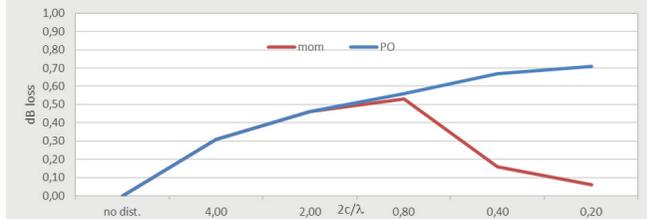
Node spacing ns	Correlation distance	MoM solution		PO solution		Equivalent Ruze δ_{rms}
		Directivity dBi	Loss dB	Directivity dBi	Loss dB	
mm	$2c/\lambda$	Directivity dBi	Loss dB	Directivity dBi	Loss dB	λ
no dist.	-	23.46	0.00	23.45	0.00	
1.20	4.00	23.15	0.31	23.14	0.31	0.043
0.60	2.00	23.00	0.46	22.99	0.46	0.052
0.24	0.80	22.93	0.53	22.89	0.56	0.057
0.12	0.40	23.30	0.16	22.78	0.67	0.063
0.06	0.20	23.31	0.15	22.73	0.72	0.065

The same random errors as for the previous plane wave illumination with a peak value of 0.04 mm are used in this case. The resulting peak directivities are shown in columns 3 and 5 of **Table 4** for the MoM and the PO solutions, respectively. The loss relative to the non distorted surface is presented in columns 4 and 6.

As for the plane wave illumination the RF results show that for the node spacings $ns = 1.2$ mm and 0.6 mm, where the diameter of Ruze's correlation region $2c$ is larger than the wavelength, the agreement between MoM and PO is very good. Changing towards a more rapidly varying surface distortion by decreasing the

node spacing, specifically $ns = 0.24, 0.12,$ and 0.06 mm, the MoM and PO solutions start to deviate and for the most rapidly varying surface distortions, $ns = 0.06$ mm, the convergence of the PO solution is simply not possible, see **Figure 15**. Furthermore, as in the plane wave illumination case, the peak directivity of the MoM result starts to increase when the correlation distance becomes less than a wavelength, and for $ns = 0.06$ mm the loss relative to the smooth surface is only 0.05 dB.

From the losses in the PO calculation an equivalent value of the aperture rms error, δ_{rms} , can be found using the Ruze equation. Notice, that the large taper

Figure 15 – Peak loss as a function of the surface roughness, $\delta_{rms} = 0.064$ mm

of the Gaussian beam introduces a weight in the rms function of the surface roughness giving different rms values for different correlation lengths. The value in column 6 corresponds very well to the rms error inside a surface region of $1.2 \cdot w_0$.

We conclude that Ruze's formula must be corrected for realistic tapered illuminations, which can be done by applying a weighting, similar to the tapering, to the rms value of the distortions. The MoM should be used over PO for $2c < 0.8\lambda$.

Conclusion

For the random surface model in GRASP, defined by the node spacing ns and the peak value ϵ_p , the relations to the right holds true.

If the aperture error $\delta_{rms} > 0.1\lambda$, Ruze's equation for gain loss is inaccurate.

If the diameter of the correlation region $2c < \lambda$, both PO and the Ruze gain loss equation give too large values for plane-wave illumination and MoM must be used to calculate the radiation pattern.

Ruze's formula must be corrected using a weighted rms value for the distortion for realistic tapered illuminations, while PO can be used for $2c > 0.8\lambda$. For smaller values the MoM should be applied.

Random surface rms error: $\epsilon_{rms} = 0.47\epsilon_p$

Correlation length: $L_c = 0.77 ns$

Diameter of correlation region as defined by Ruze: $2c$

Aperture rms error for offset reflector: $\delta_{rms} = (1 + \cos(\nu)) \epsilon_{rms}$

The Ruze gain loss as a function of the aperture error: $10 \log e^{-\left(\frac{2\pi \delta_{rms}}{\lambda}\right)^2}$

List of References

- [1] Knud Pontoppidan, "Reflector Antennas - An overview of Surface Distortion Effects," in *Proceedings of JINA '86*, Nice, Nov. 1986, pp. 82–93.
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