Robust Optimization for Shaped Reflector Antennas

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***Abstract*—The shaping of reflector antennas in geostationary orbit to attain contoured beams on earth is most often done by solving a minimax optimization problem based on a Physical Optics analysis of the reflector antenna gain, cx-pol, e.t.c. Unfortunately, this optimization problem can be highly non-linear, leading to slow convergence and many local minima, particularly for poor starting guesses.**

In this paper, it is shown how better designs that more closely fulfill the optimization goals and are more stable towards poor starting guesses can be obtained by preceding the minimax optimization with the solution of another, closely related, optimization problem.

I. Introduction

Design of antennas intended to provide a contoured beam from a geostationary orbit requires a numerical optimization to provide strong performance. Optimization is used to produce the desired coverage by means of surface shaping, excitation coefficients, feed orientation, or a range of other possible optimization variables. The goal of the optimization is to produce an antenna configuration that meets one or several specified goals on the performance in the coverage, e.g., ensuring a high gain or low crosspolar levels at specific areas in the coverage.

The performance of the antenna system can be simulated by a range of methods, but for reflector antenna systems the Physical Optics (PO) method, possibly augmented with the Physical Theory of Diffraction (PTD), is the typical choice. With PO/PTD, it is possible to optimize the antenna design by evaluating the gain, the cross-polar performance, and other relevant quantities across the coverage region as the optimization variables are modified, and then letting the optimization algorithm perform the search for the best possible value of the goal function.

A common way to define the goal function is to consider the worst-case performance of the antenna system, i.e. the largest deviation from the specified goals. This leads to a so-called minimax problem, which seeks to produce the system with the best worst-case performance; see e.g. [1]. It is a nonlinear optimization problem which means that it is generally too computationally expensive to solve it to global optimality. Thus, in practice, local optimization is used to find a local minimum. This approach often depends on the antenna designer providing a suitable initial design as a starting point for the local optimization, a task that can be notoriously difficult even for experienced designers. Unfortunately, the quality of the design obtained via the minimax approach is sensitive to the initialization.

To address the sensitivity of the minimax approach to the starting point, we consider a different measure of the performance of a particular design. The result is a one-sided nonlinear least-squares (OLS) problem in which the cost function takes all points where the goal is not met into account instead of only the worst point. The resulting problem has some interesting properties that makes it less susceptible to local minima, as has been discussed previously in the literature [2]. In this paper, we will focus on the use of the OLS problem as a means to find an initial design for the minimax approach.

II. Problem Formulation

We start by defining m functions $f_1(x), \ldots, f_m(x)$ where $f_i(x) : \mathbb{R}^n \to \mathbb{R}$ represents the performance of the antenna system (simulated e.g. by PO/PTD) associated with the ith position in the coverage region as a function of a vector x of n design variables. For each of the m positions, we have a given performance goal g_i, and we include weights w_i that may be used to emphasize important goals or indicate minimization or maximization. These quantities are then combined to yield a residual function

$$r_i(x) = w_i(g_i - f_i(x)) \tag{1}$$

for each of the m positions. The elements of x may represent the parameters in a shaped surface, the position or excitation of feeds and arrays, etc.

With the definition of the residual functions given in (1), the minimax antenna design problem can be expressed as

$$\begin{array}{ll} \underset{x \in R^n}{\text{minimize}} & \max_i r_i(x) \\ \text{subject to} & Cx + d \geq 0. \end{array}$$

This objective function is convex, but it is not everywhere differentiable, and its piecewise behaviour caused by changing residual functions governing the objective function lead to an often complicated behaviour, particularly far from an optimum. As an alternative to the max objective, we proposed in [2] a one-sided least-squares (OLS) objective of the form

$$\begin{array}{ll} \underset{x \in R^n}{\text{minimize}} & \sum_{i=1}^m \max\{0, r_i\}^2 \\ \text{subject to} & Cx + d \geq 0. \end{array}$$

Unlike the max objective function, the OLS objective function is continuously differentiable, and since it includes all non-fulfilled residual functions and is locally quadratic, the behaviour of the objective function is less challenging than for the minimax formulation.

A. Numerical implementation

We now provide a brief overview of an algorithm for local minimization of the OLS problem, described in detail in [2].

By introducing auxiliary variables, we apply the epigraph formulation to arrive at the equivalent problem

$$\begin{array}{ll} \underset{x\in R^n, u\in R^m}{\text{minimize}} & \|u\|_2^2 \\ \text{subject to} & Cx+d\geq 0 \\ & u_i \geq r_i(x), \qquad i=1,\dots,m \\ & u_i \geq 0, \qquad\quad i=1,\dots,m. \end{array} \tag{2}$$

The problem has a convex quadratic objective function, but the inequality constraints $u_i \geq r_i(x)$ may be non-convex.

To minimize (2) locally, we propose to use a trust-region method. This requires a model of the problem that serves as a surrogate within a trust-region. At the kth iteration, we obtain a convex model by linearizing the residual functions $r_i(x)$ around x^k, resulting in the trust-region problem

$$\begin{array}{ll} \underset{\Delta x\in \mathbb{R}^n, u\in \mathbb{R}^m}{\text{minimize}} & \|u\|_2^2 \\ \text{subject to} & C\Delta x+d^k\geq 0 \\ & u \geq r^k + J^k \Delta x \\ & u \geq 0 \\ & \delta^k \geq \|\Delta x\|_\infty , \end{array} \tag{3}$$

where $\delta^k > 0$ is the trust-region radius, and

$$d^k = Cx^k + d, \quad r^k = r(x^k), \tag{4}$$

$$J^k = \left[\nabla r_1(x^k) \ \dots \ \nabla r_m(x^k)\right]^T . \tag{5}$$

We use an interior-point method [3] to yield a step Δx.

III. Results

To illustrate the effect of the initialization strategy, we consider a case where the objective is to shape a geostationary reflector system for maximum gain over a CONUS coverage (continental United States). The surface of the reflector is parametrised using $n = 783$ spline variables using the TICRA software POS, and the coverage is discretized at $m = 810$ points.

The initial guesses, used as a starting point for the optimization, were created by de-focusing a paraboloidal reflector such that the radiated main beam covers the coverage region. However, moving from guess 1 to guess 6 in Table I, the de-focusing was performed on a progressively shifted coverage, such that guess 1 is expected to produce the best results and initial guess 6 is expected to produce the worst results. This is intended to highlight the performance in scenarios where strong starting guesses cannot be obtained. We did not impose constraints on x. An example of the coverage obtained with the minimax algorithm initialized with initial guess 2 is shown in Fig. 1.

In our experiments, we limited the number of function evaluations ($k_{\max}$) to 500, and for each of the six initial guesses we did two experiments. In the first experiment, we applied the OLS algorithm for 250 iterations (or until a stopping criteria is met), followed by the minimax algorithm for the remaining iterations. In the second experiment, we used only the minimax algorithm for 500 iterations. Our results are summarized in Table I; recall that smaller residuals are better. The results clearly show that the initialization strategy can improve the minimum gain in the coverage: the gain is improved for initial guesses 2-6, and the result remains the same for initial guess 1. Considering the maximal residual — it measures how far we are from reaching the goal value — the absolute improvement is largest for initial guess 6 (1.67 dB), where there is much room for improvement, and the relative improvement is largest for initial guess 3 (49.5%).

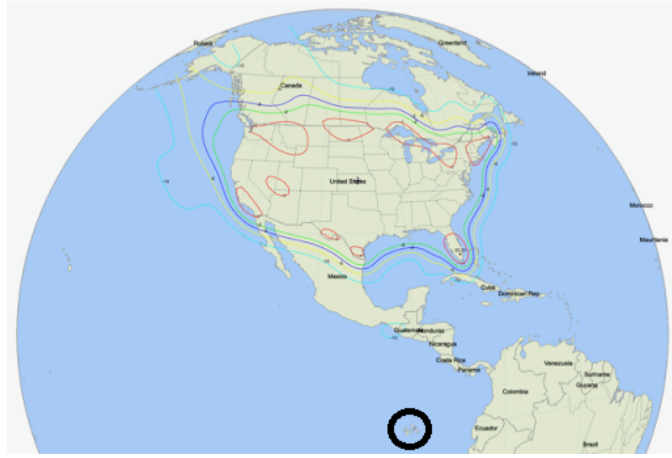

Fig. 1. Contours of coverage optimized using the minimax algorithm. The peak gain of around 33 dB is at the cross in Florida. The Nadir direction of the satellite is shown in the black circle.

TABLE I. Value of Maximal Residual in dB After 500 Iterations. The 6 Initial Guesses Become Progressively Worse: 1 is a Good (Green) Initial Guess and 6 is a Poor (Red) Initial Guess.

Initial Guess	1	2	3	4	5	6
Minimax only	-0.074	0.090	0.826	2.019	4.149	9.360
OLS+minimax	-0.074	-0.004	0.417	1.952	4.103	7.690
Difference	0.000	0.094	0.409	0.067	0.046	1.67

IV. Conclusion

The use of one-sided least-squares for initializing the minimax algorithm can lead to improved results: in our numerical experiments, the initialization strategy led to an improved design with five out of six initial guesses.

References

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