Design of High-Performance Antenna Systems with Quasi-Periodic Surfaces

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***Abstract*—Periodic surfaces (identical array elements) have existed for several decades. However, in recent years, there is an interest in replacing such surfaces with quasi-periodic surfaces (non-identical array elements). Quasi-periodic surfaces provide additional degrees of freedom and can be used to enhance existing antenna solutions or to provide new innovative concepts for future applications. Quasi-periodic surfaces usually consist of several thousands of array elements, thus an appropriate and efficient synthesis is a challenging task. In this paper, we present a general design framework for the design of high-performance antenna systems with quasi-periodic surfaces. The design framework is based on the use of accurate analysis methods used in conjunction with a large-scale direct optimization approach where all array elements in the quasi-periodic surface are optimized simultaneously in their final operational environment to directly fulfill certain pattern specifications. Using the proposed design approach, quasi-periodic surfaces that consist of more than tens of thousands of array elements can be optimized.**

***Index Terms*—Reflectarrays, frequency selective surfaces, quasi-periodic surfaces, optimization, space applications**

I. Introduction

Periodic surfaces [1] that can either reflect or transmit electromagnetic fields when illuminated by an external source have existed for several decades. Such surfaces include frequency selective surfaces (FSS), polarisation selective surfaces, etc. The RF-design of such periodic surfaces is usually done at the unit-cell level where an infinite array consisting of identical unit-cells illuminated by a plane wave is assumed. The unit-cell is then optimized to fulfill a set of reflection and transmission specifications from which the final design is obtained. There are significant drawbacks associated with this approach. The finite size of the surface is not accounted for during the optimization. Plane wave illumination is assumed and the actual dependence of the amplitude and phase of the illuminating field are not taken into account. These factors may result in designs that has suboptimal performance when the periodic surface is exposed to a large range of incidence angles, which is often the case in practical designs. Therefore, there is interest in replacing periodic surfaces using quasi-periodic surfaces using a design methodology similar to reflectarrays so as to maintain good angular performance.

For quasi-periodic surfaces such as reflectarrays [2], the conventional design process involves an additional step after the design of the unit-cell. In this step, each element on the surface is optimized, a single element at the time, to arrive at the final design. The drawback of this method is that one is optimizing one element at the time and that it is an indirect optimization approach where you do not maintain the direct relation between the optimization variables (geometry) and the optimization goals (near/far-field pattern), thus leading to suboptimal designs.

The antenna community is constantly investigating antenna solutions that use quasi-periodic surfaces to either improve existing solutions or to provide new innovative concepts for future applications, e.g., [3]–[5]. In [3], an FSS that operates in 20/30 GHz is used in an dual-reflector configuration to separate the two bands. A low-profile curved quasi-periodic surface that operates in 20/30 GHz is proposed in [4]. And in [5], doubly curved reflectarrays are considered to compensate for beam squint in orthogonally polarized Tx/Rx offset antenna systems. For all these concepts, a dedicated but general design framework is needed to ensure designs with good performance. Furthermore, the design tool must be able to handle quasi-periodic surfaces that consist of more than tens of thousands of array elements.

In this work, we present a general design framework that can be used for the design of high-performance antenna systems consisting of quasi-periodic surfaces.

II. Analysis of Periodic Unit-Cells

The first thing that is needed in a general design tool is the possibility of analyzing the geometry of the unit-cells that is used in the quasi-periodic surface. A general purpose code, e.g., HFSS, has the advantage that it can handle any geometry, however, at the expense of computational speed. In our case, we have developed three different periodic solvers, two dedicated and one general. This is to maintain the generality but at the same time to ensure that the an efficient solver can be applied to the specific geometry.

All three algorithms are surface integral equation (IE) methods. The geometry is arranged in a 2D lattice (skewed lattice defined by two vectors) and analyzed assuming that it is located in an infinite array consisting of the same geometry.

A. Spectral Domain Periodic MoM

Printed multi-layered structures are commonly used in quasi-periodic surfaces. For these type of surfaces, a dedicated higher-order spectral domain method of moment (SDMoM) has been developed. The periodic problem is formulated in terms of an IE and solved in the spectral domain [6]. The

Green's function in the IE consists of a double summation of Floquet harmonics. In the SDMoM, one can either discretize unknown electric surface currents (patch type elements) or unknown electric aperture fields (aperture type elements). The SDMoM can easily handle many dielectric layers, it is efficient and well-validated, however, the metallization layers must be confined to the interfaces between the dielectric layers.

B. Mode-Matching Periodic MoM

Periodic perforated structures are often used in dichoric plate filters. For such structures, an IE solution based on a mode-matching approach similar to that presented in [7] has been developed. First the method expands the field inside the perforations/apertures in characteristics modes similar to the modes in a wave guide. To this end, a finite element method is used. Then by matching these waveguide modes to appropriate periodic boundary conditions (Floquet harmonics), one can solve the periodic problem to determine the unknown electric aperture fields.

C. Spatial Domain Periodic MoM

The last solver is a periodic MoM suitable for general 3D objects or structures that that can not be analyzed using the algorithms mentioned above. The method operates in the spatial domain using a periodic free-space Green's function which is accelerated using the Ewald's method. Our formulation follows that of [8], however, there are several major extensions. First, it is a higher-order formulation, hence leading to faster solution with fewer unknowns. Second, our formulation works with dielectrics and composite metallic/dielectric structures. Finally, our method employs a discontinuous Galerkin approach which allows non-connected meshes while still preserving high accuracy.

III. Analysis of Quasi-Periodic Surfaces

The methods described in Section II are periodic method of moments (PMoM) solvers that analyze a single periodic unit-cell. We now turn our attention on to the analysis of finite sized quasi-periodic surfaces. To this end, two methods have been developed and described below.

A. PMoM/PO Algorithm

In this method, each array element is analyzed assuming local periodicity, i.e., the individual element is assumed to be located in an infinite array of identical elements [9]. The reflection/transmission characteristics of the each element are determined by any of the PMoM solvers mentioned in Section II and are subsequently used to form equivalent currents from which the far-field is calculated. The equivalent magnetic and electric currents are defined by

$$\boldsymbol{M}_\mathrm{S} = -\hat{n} \times \boldsymbol{E}, \quad \boldsymbol{J}_\mathrm{S} = \hat{n} \times \boldsymbol{H}, \tag{1}$$

respectively, and constructed on a surface that encloses the finite sized surface. Herein, $\boldsymbol{H}$ and $\boldsymbol{E}$ are the total magnetic and electric fields at the surface and $\hat{n}$ is the outward unit vector normal to that surface. For structures with a PEC ground plane, e.g., reflectarrays, the total field in the entire half space behind the quasi-periodic surface is assumed to be zero hence equivalent currents are only constructed over the top surface of the structure.

Since each individual element is analyzed using a PMoM solver and the equivalent currents are generated similar to the PO approximation, this method is denoted the PMoM/PO method.

B. PO Algorithm

For a perfectly conducting surface, the PO approximation consists of finding the equivalent electric surface currents from $\boldsymbol{J}_\mathrm{S} = 2\hat{n} \times \boldsymbol{H}^\mathrm{i}$ with $\boldsymbol{H}^\mathrm{i}$ being the incident magnetic field. For quasi-periodic surfaces, rather considering each individual array elements as it is the case using the PMoM/PO method, we can consider the surface as a continuous modulated surface impedance. By doing so, we remove any references to the individual array elements. By applying the equivalence principle, equivalent currents enclosing the finite sized surface is defined as in (1).

The modulation of the impedance over the surface is generated by varying the geometry of the array elements. For a given geometrical parameter of the array element, a, the variation over the surface is expressed in terms of basis splines as function of the position

$$a(x,y) = \sum_{i}^{N_i} \sum_{j}^{N_j} c_{ij} B_i(x) B_j(y), \tag{2}$$

where c_{ij} are the spline coefficients, $B_i(x)$ and $B_j(y)$ are spline functions, and N_i and N_j the number of spline functions in x and y, respectively. By varying c_{ij}, the modulation over the surface is varied. The reflection and transmission coefficients which are needed to compute (1) at any given point on the surface are calculated using one of the PMoM solvers mentioned in Section II.

Although PMoM is used, since there are no references to the individual array elements, this method is denoted the PO method.

IV. Curved Quasi-Periodic Surfaces

FSS and traditional quasi-periodic surfaces such as reflectarrays are usually planar. However, curved quasi-periodic surfaces are gaining interest due to an additional degree of freedom which allows the use of innovative concepts. For instance, all the concepts considered in [3]–[5] make use of a curved quasi-periodic surface. Consequently, it is important that the general design framework can treat such curved surfaces.

A. PMoM/PO Algorithm

For the analysis of curved quasi-periodic surfaces using the PMoM/PO method, an equivalent configuration has to be defined to approximate locally the curvature of the surface.

The actual configuration under consideration is shown in Fig. 1a. Here, we have a curved quasi-periodic surface consisting of two metallization layers. To approximate locally

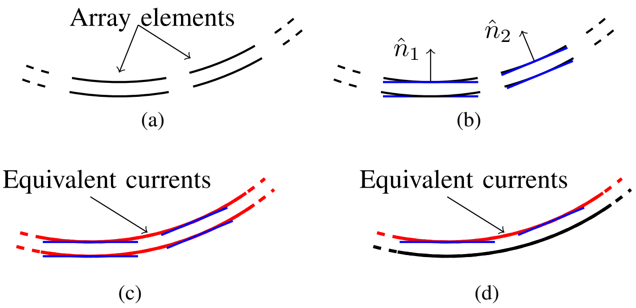

Fig. 1. Description of how PMoM/PO is applied on curved surfaces.

the curvature of each array element, an equivalent planar configuration is defined. This is shown in Fig. 1b where each array element is assumed to be locally planar as shown with solid blue lines. The normal vectors $\hat{n}$ in Fig. 1b are used to determine the angle of incidence for each array element. Each element is than analyzed using a PMoM solver from which the reflection and transmission coefficient for each array element are calculated. Using these coefficient matrices, equivalent currents are defined on the top and bottom surface as shown in Fig. 1c with the red lines. The equivalent currents are defined on the curved surface, thus taking into account the actual curvature. From these equivalent currents, the far-field of the quasi-periodic surface can be calculated.

For quasi-periodic surfaces with a ground plane, e.g., reflectarrays, the equivalent currents are only defined at the top surface as illustrated in Fig. 1d.

B. PO Algorithm

The treatment of curved surfaces using the PO method is straightforward. The equivalent currents are simply calculated at predefined sampling points (depending on the quadrature rule that is used for the surface integrals) on the top and bottom surface of the curved surface. At each sampling point, the incident angle is determined from the vector normal to the surface at that point and used in the PMoM solver to obtain the equivalent currents. For surfaces with a ground plane, the equivalent currents are only defined at the top surface.

V. Optimization of Quasi-Periodic Surfaces

For reflectarrays, using a direct optimization approach where all array elements are optimized simultaneously to fulfill the pattern specifications can provide enhanced performance compared to in-direct optimization approaches [10]. The fact that all elements are optimized simultaneously in a direct manner implies that a local mismatch between the desired and actual element performance can be compensated by all other elements. This is not possible when the elements are optimized one-by-one which is often the case for the design of quasi-periodic surfaces in the literature. Therefore, the same direct optimization approach is adopted here for general quasi-periodic surfaces.

A. Optimization Algorithm

The optimization algorithm is a gradient-based non-linear minimax algorithm and the general framework is taken from [11]. The main problem with the algorithm from [11] is that it is not well suited for solving large-scale optimization problems that arise in the direct optimization of all array elements in an electrically large quasi-periodic surface where the number of variables can exceed tens of thousands. In many optimization cases, the problem was unsolvable using the original algortihm due to computational constraints. Consequently, a number of improvements have been done to make it suitable for large-scale optimization problems.

Most notably, an interior-point solver is used to find the steepest descent feasible direction in the optimization using large steps. By taking large steps that activate multiple functions and constraints each time, a faster and easily parallelized implementation can be achieved.

Furthermore, the constraint matrix is often extremely sparse. The algorithm in [11] cannot exploit sparsity due to the use of a non sparsity-preserving factorization, thereby consuming significant amount of memory just to store the constraint matrix. In the new implementation, sparsity can easily be exploited thereby reducing the memory footprint.

B. Optimization Approaches

When doing the direct optimization of quasi-periodic surfaces, two approaches can be used depending on the application at hand. One can use the PMoM/PO as the analysis method in the optimization, or one can use the PO method.

When using the PMoM/PO as the analysis method in the direct optimization, the optimization variables are the adjustable parameters of array elements. For instance for a rectangular patch, the parameters could be the length and the width of the patch. All array elements are optimized simultaneously to fulfill certain optimization goals. Consequently, the total number of optimization variables is $N_{\text{total}} = N_{\text{el}} N_{\text{geo}}$, where N_{el} and N_{geo} are the number of array elements and the number of adjustable parameters per element, respectively. It is clear that for a quasi-periodic surface consisting of tens of thousands of array elements, where each element has several adjustable parameters, the total number of optimization variables will be large and the optimization may be challenging.

The advantage of optimizing quasi-periodic surfaces using the PMoM/PO method is that it maintains the available degrees of freedom provided by all the array elements. This is particularly important when designing quasi-periodic surfaces such as transmitarrays and reflectarrays where the variation of the geometry of the array elements over the surface can have discontinuities where the phase of the reflected field is required to jump after a complete 360°. The drawback is the computational complexity imposed by the number of array elements and the number adjustable parameters.

When using the PO as the analysis method in the optimization, one can not optimize each individual array element since there is no reference to the individual array elements. Instead, the optimization variables are the spline coefficients c_{ij} in (2).

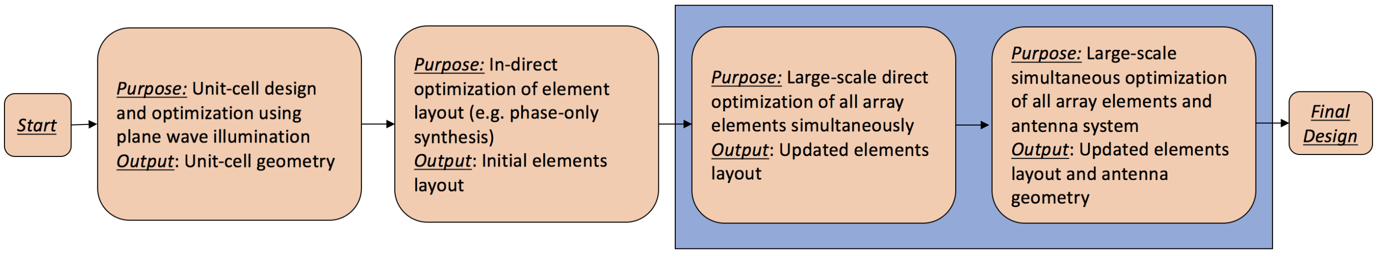

Fig. 2. Procedure for the design of quasi-periodic surfaces. The initial steps in the design process are identical to the existing design methods. The steps in the blue box are additional steps which involve large-scale direct optimization of all array elements simultaneously.

By optimizing the coefficients, one alters the geometries of the array elements over the surface. To extract the actual geometry of the elements after an optimization, the spline coefficients of the geometrical parameters are sampled at the positions of the actual element.

The advantage is that the number of optimization variables is not dictated by the number of array elements but rather the electrical size of the quasi-periodic surface. The larger the size, additional spline functions should be used to describe the variation over the surface. However, this advantage is acquired at the cost of the limitations imposed by the spline functions used to describe the surface. Splines are continuous functions that cannot effectively represent discontinuities in the variation of the geometry of the array elements over the surface which are often required in planar reflectarrays or transmitarrays, thus it is not suitable for the design of such surfaces. However, for surfaces where a smooth geometrical variation is expected, e.g., an aperiodic FSS where the elements are optimized to compensate for the various incidence angles [12], this approach is preferred.

To summarize, the method used during the optimization should be dictated by the application and geometry under consideration. For surfaces where a smooth geometrical variation is expected, the PO method should be used, whereas if discontinuities in the geometrical variation is called for, the PMoM/PO method is favoured.

VI. Design of Quasi-Periodic Surfaces

The design of a quasi-periodic surface is done in several steps as illustrated in Fig. 2. The first steps in the process corresponds to the conventional design process of quasi-periodic surface. First, the unit-cell is optimized to fulfill a set of reflection and transmission specifications. Subsequently, each element on the surface is optimized, a single element at the time.

Once these initial steps are completed, additional optimization steps are performed to enhance the performance as shown in the blue box in Fig. 2. First, using the design obtained in the initial steps as the starting point, the direct optimization of all array elements are performed from which an updated elements layout is obtained. As a final step, all the array elements can be optimized together with the remaining part of the antenna system simultaneously. This gives the possibility to optimize the quasi-periodic surfaces in their final operational environment by also allowing reflectors and other components to be included in the optimization.

This also implies that the quasi-periodic surfaces can be optimized for the secondary pattern performance which is necessary in certain applications to ensure good performance. An example of this could be a dual-reflector antenna where the subreflector is a quasi-periodic surface. Then the quasi-periodic surface could be optimized for the pattern performance of the main reflector.

VII. Application Example

To demonstrate the capabilities of the design framework, we consider here an application example taking outset in the concept proposed in [4]. The concept considers a quasi-periodic surface that provides dual-band linear-to-circular polarization with the interesting property that the same linear polarization (LP) is converted into a given circular polarization (CP) handedness over the first frequency band and into the orthogonal one over the second frequency band. This feature is of interest for HTS applications in Ka-band (Tx/Rx at 20/30 GHz) as it can be used to reduce the number of antennas.

The antenna configuration we consider is a standard offset reflector with a circular projected diameter of 1.35 m and a focal length of 1.4 m. The polarizing element is a simple rectangular patch (dipole) printed over a single layer substrate ($\epsilon_r = 3.5$, $\tan\delta = 0.0035$, thickness (h) of 1.524 mm) and backed by a ground plane [4]. To achieve the desired LP-to-CP conversion in Tx and Rx, the dipole has been optimized in [13] to have a length of $L = 5.13\,\text{mm}$ and a width of $w = 0.15\,\text{mm}$ with a cell-size of $9.32 \times 0.42\,\text{mm}^2$.

This element is distributed periodically over the surface of the reflector and is oriented such that the x-dimension (the narrower dimension) of the cell is along the offset plane. This is to take advantage of the offset geometry such that the worst scan angular range corresponds to the polarizer's plane, which is less sensitive to the incidence angle. The polarizing reflector is illuminated by a linearly polarized Gaussian beam.

An analysis of the polarizing antenna at 19.7 GHz and 29.5 GHz shows that a RHCP beam is radiated at 19.7 GHz with a peak gain of 47.8 dBi and a LHCP beam at 29.5 GHz with a peak of 50.9 dBi. This confirms the operation of the LP-to-CP conversion. However, the cross-polarzation level is around 20 dBi and 35 dBi at 19.7 GHz and 29.5 GHz, respectively, which is too high, in particular in the Rx band.

To reduce the cross-polar radiation, the direct optimization is applied. Due to the small size of the dipoles, the number of array elements on this reflector exceeds 300,000. Furthermore, the array elements are optimized to compensate for the various incidence angles, thus the geometry variation over the surface is excepted to be smooth. Thus, for this application the PO method is the preferred analysis method used during the optimization. As optimization variables, L and w of the polarizing elements are included. To represent the geometry variation, 24×24 spline functions were sufficient for each parameter yielding a total number of 1,152 optimization variables. As optimization goals, the peak gain should be maintained at least 47.0 dBi and the cross-polar radiation should be suppressed as much as possible in both Tx and Rx. The entire optimization took less than a half hour on a standard laptop computer.

The performance of the optimized polarizing reflector is shown in Fig. 3 together with the patterns of the original reflector. It is seen that the cross-polar radiation in 29.5 GHz has been significantly reduced with approximately 10 dB, now with a cross-polar level around 26 dBi. This is obtained at the cost of higher cross-polar radiation at 19.5 GHz, now with a cross-polar level around 23.5 dBi. This is however expected since no weighting between the two bands was specified in the optimization.

It is worthwhile to note that the critical point during the optimization is the performance of the polarizing reflector in the Rx band. If only the Tx band is considered in the optimization, the cross-polar radiation can be suppressed to an XPD better than 30 dB. This cannot be achieved if only the performance is optimized at the Rx band. This points to the fact that a better polarizing cell which is more angular insensitive is needed en ensure optimal LP-to-CP conversion.

VIII. Conclusions

This paper presents a general design framework that can be used for the design of high-performance antenna systems consisting of quasi-periodic surfaces. The main feature is that it combines dedicated methods for the analysis of finite sized quasi-periodic with an efficient large-scale optimization algorithm. This allows the direct optimization of electrically large quasi-periodic surfaces consisting of tens of thousand of array elements in their final operational environment. The design approach can be applied on various antenna systems consisting of quasi-periodic surfaces and provides with advanced capabilities otherwise not possible.

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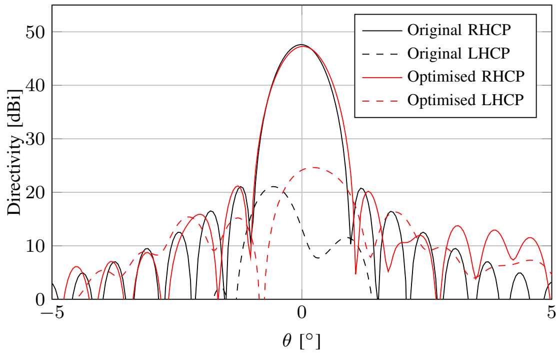
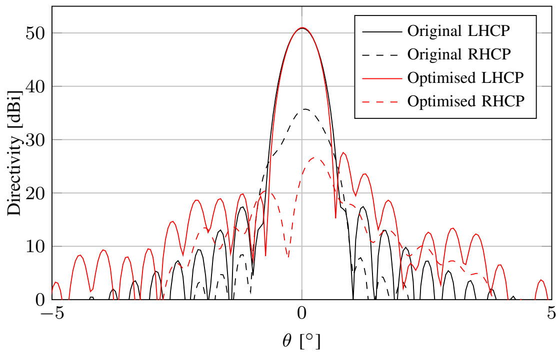


(a) 19.7 GHz



(b) 29.5 GHz

Fig. 3. Performance of the optimized LP to CP polarizing reflector.

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