LENS ANALYSIS METHODS FOR QUASIOPTICAL SYSTEMS

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Abstract

The Method of Moments (MoM), Physical Optics (PO) and Geometrical Optics (GO) are compared for analysis of dielectric lenses. It is found that MoM can be used (with normal computers) for diameters of the lens up to 15λ, λ being the wavelength, whereas the approximate methods PO and GO must be used at higher frequencies. Good agreement between the different methods is found and efficient calculation of PO and GO is discussed.

1 Introduction

Lenses have become important as focusing or phase-correcting elements in the quasioptical frequency range from approximately 300 GHz to 1 THz. Beam waveguides in this frequency range are normally designed by means of Gaussian beams, but often the dimensions of the components are too small (in wavelengths) to rely solely on optical methods or Gaussian beam analysis. In this paper the Method of Moments (MoM) is compared to Physical Optics (PO) and Geometrical Optics (GO) for analysis of dielectric lenses.

2 Lens analysis theory

Analysis of dielectric lenses using MoM, PO and GO will be explained in the following subsections. The methods are all based on the equivalence principle which allows the scattered field to be represented by the radiation of equivalent currents flowing on the surfaces of the lens. Rotational symmetry is not assumed for any of the methods.

2.1 Equivalence principle for lens analysis

In Figure 1 a horn radiates a field $\mathbf{E}_i$, $\mathbf{H}_i$ in region 1 in a medium characterized by the dielectric constant ε_1. This field is scattered by a homogeneous dielectric object that occupies region 2 and has a different dielectric constant ε_2. The resulting total field in the two regions is denoted **E**, **H**. In the MoM theory it is customary to represent the scattered field by a set of equivalent electric (**J**) and magnetic (**M**) currents flowing on the interface between region 1 and 2, [1].

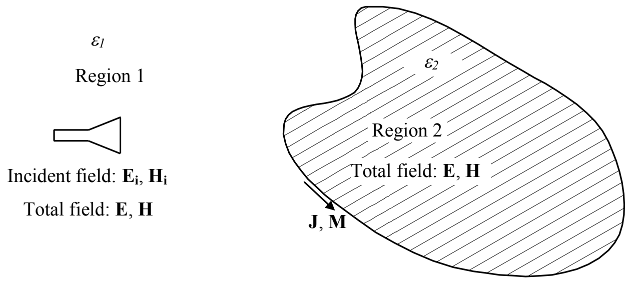

Figure 1: Equivalent currents on dielectric object.

The usual choice of the currents **J** and **M** is such that if all space is filled by a medium of dielectric constant ε_1, the currents will radiate the field $\mathbf{E}$-$\mathbf{E_i}$, $\mathbf{H}$-$\mathbf{H_i}$ in region 1 and the field -$\mathbf{E_i}$, -$\mathbf{H_i}$ in region 2. If, on the other hand, all space has a dielectric constant of ε_2 the same currents will radiate a zero field in region 1 and the field -**E**, -**H** in region 2. If the equivalent currents are known, it is thus possible to calculate the total field everywhere, both outside and inside the lens. It can be shown that the currents are related to the total field on the boundary between region 1 and 2 by

$$\mathbf{J} = \hat{\boldsymbol{n}} \times \mathbf{H}\,, \qquad \mathbf{M} = -\hat{\boldsymbol{n}} \times \mathbf{E}\,, \qquad (1)$$

where $\hat{\boldsymbol{n}}$ is the outwards pointing unit normal vector. The purpose of all the following analysis procedures is to calculate the equivalent currents, such that the total field can be found everywhere.

A lens located in free space is now considered, as shown in Figure 2.

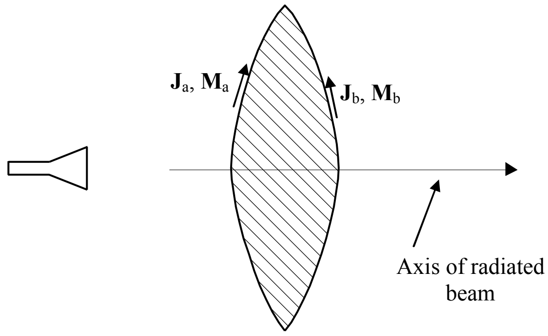

Figure 2: Equivalent currents on lens.

The equivalent currents are separated in the currents $\mathbf{J}_a$, $\mathbf{M}_a$ on the left hand surface and the currents $\mathbf{J}_b$, $\mathbf{M}_b$ on the right hand surface of the lens. The total field is found by integrating the currents on both surfaces and adding the incident field from the horn. It often happens, however, that the support structure that keeps the lens in place is large and blocks any spill-over around the lens. A lens mounted in an infinite conducting screen will then be a better model, as shown in Figure 3.

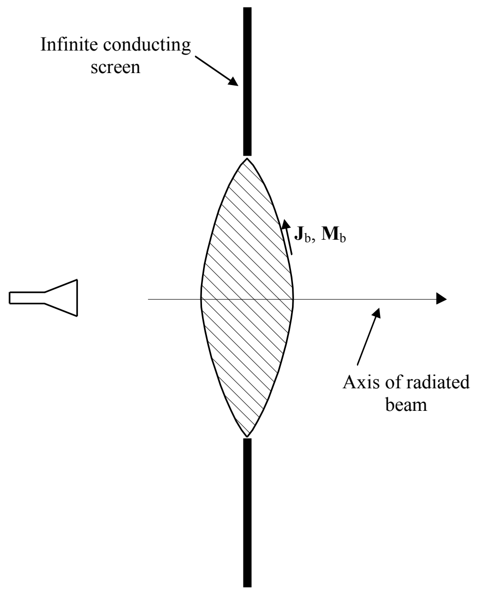

Figure 3: Lens mounted in an infinite conducting screen.

If the edge illumination of the lens is low the currents $\mathbf{J}_b$, $\mathbf{M}_b$ will not be significantly affected by the screen, and the field on the right hand surface of the conducting screen will be nearly zero. The equivalence principle then shows that the total field in the half-space to the right of the screen can be found by integrating only the currents $\mathbf{J}_b$, $\mathbf{M}_b$, excluding the radiation from $\mathbf{J}_a$, $\mathbf{M}_a$ and the incident field from the horn. In this way a good approximation to the radiation of a lens in a conducting screen can be found.

In the following subsections a number of methods for determining the equivalent currents will be described.

2.2 Method of Moments (MoM)

The Method of Moments is attractive because it is an exact method and because it is very flexible. It allows metallic support structures to be included together with the dielectrics without approximations. The drawback is that the memory requirement and computation time grow rapidly with the size of the lens. In the present study a very efficient MoM formulation is used [1] which allows analysis of lenses of up to 15λ on modern PC workstations.

2.3 Physical Optics (PO)

In the Physical Optics method it is assumed that the field on the lens surfaces (see Figure 2) behaves locally as a plane wave, which allows an approximate calculation of the equivalent currents. The first step in the procedure is to determine the currents $\mathbf{J}_a$, $\mathbf{M}_a$ on the surface illuminated by the source. In any point on this surface the equivalent currents can be computed using the Fresnel reflection and transmission coefficients for plane-wave incidence on a planar dielectric interface. The direction of incidence is determined by Poynting's vector such that the local reflected and transmitted field can be computed. When these fields are known the currents $\mathbf{J}_a$, $\mathbf{M}_a$ follows directly from eq. (1). By integration of $\mathbf{J}_a$, $\mathbf{M}_a$ in the dielectric lens material it is possible to compute the incident field on the right hand surface of the lens so that $\mathbf{J}_b$, $\mathbf{M}_b$ can be found. Again Poynting's vector and the Fresnel reflection and transmission coefficients are used. The procedure is illustrated in Figure 4. A suitable integration grid is set up on both surfaces and each current element on the left hand surface radiates onto each grid point on the right hand surface and contributes to the equivalent currents $\mathbf{J}_b$, $\mathbf{M}_b$. The integration procedure uses a polar grid combined with the Gauss-Legendre integration rule, see [2] for further details.

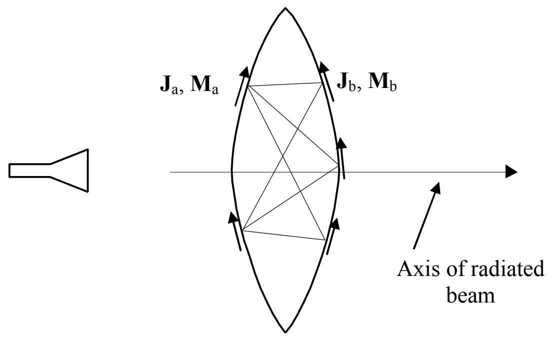

Figure 4: PO lens calculation. All currents in the integration points on the left hand surface radiate through the lens material to the integration points on the right hand surface.

As illustrated in Figure 4, the PO method includes interaction between all elements, but multiple interactions between the two surfaces are neglected. If the incident field has an irregular behaviour, the local plane-wave assumption may not be sufficiently accurate. This can e.g. happen if the lens is located close to a waist in a beam waveguide. It may then be necessary to expand the incident field in a series of plane waves as described in [3]. This will ensure an accurate calculation of the currents $\mathbf{J}_a$, $\mathbf{M}_a$, from which the field inside the lens can be computed. Also the field inside the lens will be irregular such that a second plane wave expansion is needed to compute the currents $\mathbf{J}_b$, $\mathbf{M}_b$. Although the PO method is much faster than MoM it can be time consuming to analyse lenses with diameters larger than 100λ. This is due to the small distance between the current layers which requires a dense integration grid in order to calculate the radiation from one set of currents onto the other. As shown in [4] the convergence can be improved by inserting an auxiliary plane inside the lens material.

2.4 Geometrical optics (GO)

In the GO analysis it is also the goal to compute the equivalent currents on the surfaces of the lens. Here the propagation of the field inside the lens is based on GO such that the power is conserved in ray tubes as illustrated in Figure 5. A similar procedure has been described in [5], but it will here be shown how to arrange the calculations in a simple and efficient way. Like PO, the GO analysis requires that the field behaves locally as a plane wave. The GO is more sensitive to irregular behaviour of the field than PO, but, on the other hand, it is much faster than PO.

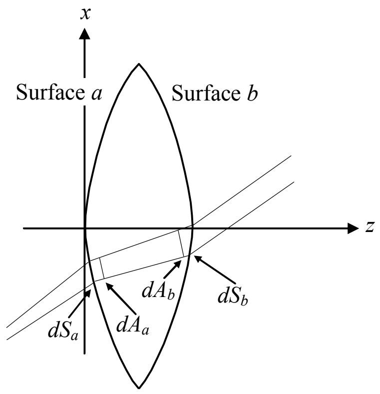

Figure 5: GO lens calculation. The power is conserved in the ray tube inside the lens.

If the field $\mathbf{E}_a$ just inside the lens on surface a is known, it is possible to calculate the field $\mathbf{E}_b$ inside the lens on surface b by the standard GO relation

$$\mathbf{E}_b = \mathbf{E}_a \sqrt{\frac{dA_a}{dA_b}} e^{-jk_2 \ell} \quad , \tag{2}$$

where the square root factor is the divergence factor that relates the amplitude of the field on surface a to the amplitude on surface b (dA_a and dA_b are the cross-section areas of the ray tube at surface a and b, respectively). The exponential factor contains the phase, where k_2 is the wavenumber $2\pi / \lambda_2$ inside the dielectrics and ℓ is the length of the refracted ray from surface a to surface b. When the incident field on surface a from the source is known, its direction of propagation is given by Poynting's vector such that the transmission coefficient of Fresnel can be applied to obtain $\mathbf{E}_a$. Hereafter, $\mathbf{E}_b$ can be found from eq. (2). The Fresnel coefficients are again used to find the field just outside the lens on surface b, and finally this field together with eq. (1) gives the equivalent currents.

A simple method to compute the divergence factor in (2) will now be described. It is convenient to parameterise surface a by the coordinates x and y in the coordinate system shown in Figure 5. The surface is given by

$$\boldsymbol{r}_a(x,y) = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z_a(x,y)\hat{\boldsymbol{z}} \quad , \tag{3}$$

where $z_a(x,y)$ is the z-coordinate of the surface. In order to use a simple forward ray tracing through the lens, the parameterisation of surface b is chosen such that $\boldsymbol{r}_b(x,y)$ is the intersection point on surface b of the refracted ray through $\boldsymbol{r}_a(x,y)$. This can also be expressed as

$$\boldsymbol{r}_b = \boldsymbol{r}_a + \hat{\boldsymbol{r}}\ell \quad , \tag{4}$$

where $\hat{\boldsymbol{r}}$ is the direction of the refracted ray given by Snell's law. In this way a backwards ray tracing is avoided which can be problematic when the incident field is not necessarily radiated by a point source.

The cross-section area dA_a of the ray tube is related to the surface element dS_a on surface a by

$$dA_a = dS_a(\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{n}}_a) \quad , \tag{5}$$

where $\hat{\boldsymbol{n}}_a$ is the inwards pointing normal on surface a. Furthermore, dS_a is given by the standard relations

$$dS_a = |\boldsymbol{N}_a| dxdy \quad , \tag{6}$$

$$\boldsymbol{N}_a = -\frac{\partial z_a}{\partial x}\hat{\boldsymbol{x}} - \frac{\partial z_a}{\partial y}\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}} \tag{7}$$

and $\boldsymbol{N}_a = \hat{\boldsymbol{n}}_a / |\hat{\boldsymbol{n}}_a|$.

In the same way it is found for surface b

$$dA_b = dS_b(\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{n}}_b) \quad , \tag{8}$$

$$dS_b = |\boldsymbol{N}_b| dxdy \quad . \tag{9}$$

From this, the divergence factor simply becomes

$$\frac{dA_a}{dA_b} = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{N}_a}{\hat{\boldsymbol{r}} \cdot \boldsymbol{N}_b} \quad , \tag{10}$$

The remaining complication is that the normal vector $\boldsymbol{N}_b$ must be computed from the general relation

$$\boldsymbol{N}_b = \frac{\partial \boldsymbol{r}_b}{\partial x} \times \frac{\partial \boldsymbol{r}_b}{\partial y}, \qquad \hat{\boldsymbol{n}}_b = \boldsymbol{N}_b / |\boldsymbol{N}_b| \quad , \tag{11}$$

where the derivatives are not known analytically, but must be found by numerical differentiation of (4), e.g.

$$\partial \boldsymbol{r}_b / \partial x \simeq (\boldsymbol{r}_b(x + \Delta x, y) - \boldsymbol{r}_b(x,y)) / \Delta x \quad . \tag{12}$$

The same integration grid can be used on both surfaces by means of (6) and (9) which relates the integration element *dxdy* to the corresponding surface elements.

2.5 Multimode Gaussian beams (MMGB)

The authors have investigated the Gauss-Laguerre multimode beam expansion in connection with beam waveguides with reflectors. The mode matching on the reflectors was done by the method described in [6]. It was found that the procedure is very fast and that it accurately describes the development of the main beam through the waveguide, but diffractions are not included. It is expected that the method has similar advantages and limitations for a beam waveguide with lenses, but the computations have not yet been carried out.

3 Examples

A plano-convex lens is used as a computational example. The geometry is defined below and the analysis is carried out by MoM, PO and GO.

3.1 Lens geometry

The plano-convex lens used in the following calculations is shown in Figure 6.

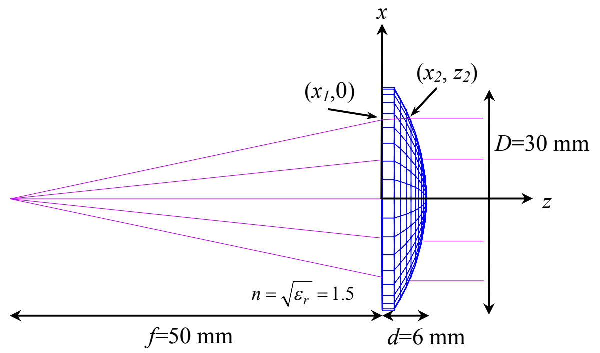

Figure 6: Plano-convex lens that transforms a spherical wave into a plane wave.

The lens is defined by the focal length f, thickness d, diameter D and the index of refraction n. It is designed by GO such that it transforms a spherical wave into a plane wave. A lens of this type is e.g. useful for correcting the phase of a corrugated horn. The actual dimensions are shown on the drawing and the curved surface is given by the following equations from [7] that relates a point on the planar surface to the corresponding point, along a refracted ray, on the curved surface.

$$z_2 = \left(\frac{(n-1)d + f - \sqrt{f^2 + x_1^2}}{n - \sqrt{1 - \frac{x_1^2}{n^2(f^2 + x_1^2)}}} \right) \sqrt{1 - \frac{x_1^2}{n^2(f^2 + x_1^2)}} \quad (13)$$

$$x_2 = x_1 \left(1 + \frac{z_2}{\sqrt{n^2(f^2 + x_1^2) - x_1^2}} \right) \quad (14)$$

In all the following test cases the incident field is an ideal Gaussian beam (complex Huygens source) with a radius of curvature of R=50 mm at the planar surface, such that the output beam will have its waist just at the right hand side of the curved surface of the lens. The width of the incident beam is w=10 mm corresponding to an edge taper of approximately -20 dB.

3.2 Lens Diameter = 10λ

The wavelength is 3 mm corresponding to a frequency close to 100 GHz. The lens is located in free space and it is seen that the general agreement between MoM, PO and GO is good, but that the 1st sidelobe is predicted too low by GO.

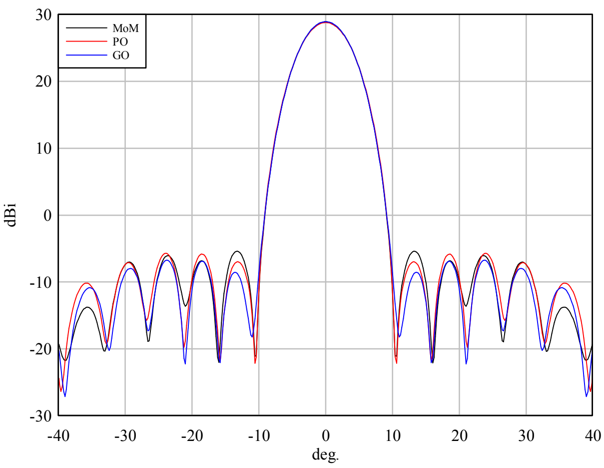

Figure 7: Comparison of MoM, PO and GO for D=10λ.

3.3 Lens Diameter = 15λ

The wavelength is now 2 mm corresponding to a frequency of approximately 150 GHz. It is seen that MoM and PO agrees very well whereas the level of the 1st sidelobe with GO is 2 dB lower.

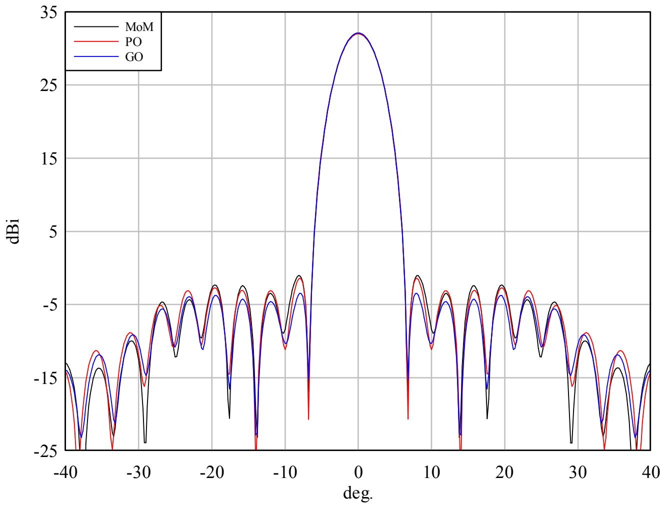

Figure 8: Comparison of MoM, PO and GO for D=15λ.

3.4 Lens Diameter = 40λ

The MoM calculations are only possible up do D=15λ on normal PC workstations, but PO and GO can be computed at much higher frequencies. Due to the perfect focusing of the lens, the patterns do not change very much (except for scaling) when the frequency is increased. As an example, the patterns for D=40λ are shown below for the lens mounted in an infinite conducting screen. It is seen that the GO sidelobes are still about 1 dB lower than the more accurate PO, but otherwise the patterns agree well.

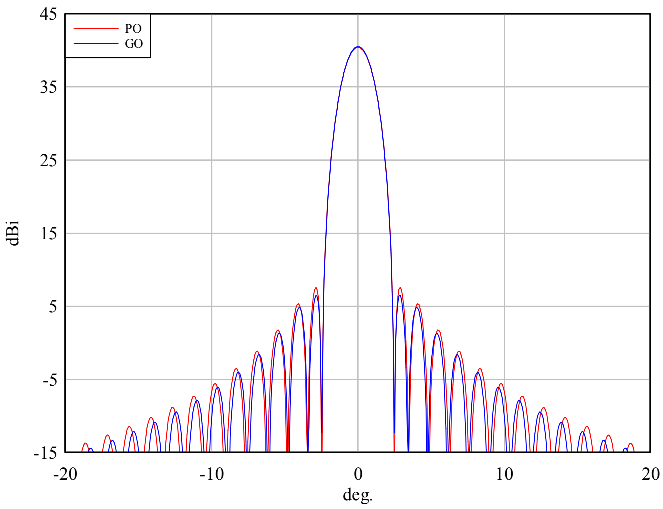

Figure 9: Comparison of PO and GO for D=40λ.

3.5 Computation time

The MoM analysis takes 8 minutes with D=10λ and 28 minutes with D=15λ on a modern PC workstation. In comparison the PO and GO analysis shown in the paper all use less that 5 seconds. For a fixed number of sidelobes the GO computation time is nearly independent of the frequency, whereas the time for PO starts to grow noticeably for D>40λ. With D=40λ the GO calculation is 3 times faster than PO, whereas it is 20 times faster for D=80λ.

4 Conclusion

It is shown that the accurate MoM analysis is useful and feasible for analysis of dielectric lenses with diameters up to 15λ with standard computers. For this size of the lens the MoM is in good agreement with the much faster PO and GO. If the size of the lens is larger than 100λ the PO calculations starts to become time consuming and the simpler and faster GO should be used. All of the analysis methods can be formulated in terms of equivalent currents which allow the field from a lens in free space to be calculated as well as the radiation from a lens in a conducting screen.

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