Efficient Body of Revolution Method of Moments for Rotationally Symmetric Antenna Systems with Offset Illumination

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***Abstract*—An efficient Body of Revolution Method of Moments formulation is presented in which the unknown surface current density on the generatrix of the rotationally symmetric scatterer is expanded in terms of higher-order hierarchical Legendre basis functions. Careful attention has been paid to fast evaluation of the involved modal Green's functions. The efficient formulation is particularly useful for the analysis of rotationally symmetric scatterers illuminated by an offset feed for which a method of moments problem must be solved for a large number of azimuthal modes.**

I. Introduction

Rotationally symmetric reflector systems are attractive candidates for realizing compact high-gain antennas with low cross polarization and low manufacturing costs. The Body of Revolution Method of Moments (BoR-MoM) is one of the preferred full-wave methods for accurate analysis of such rotationally symmetric systems. The BoR-MoM formulation is well-known and has been presented in several works, e.g., for conducting objects [1], for dielectric objects [2], and for composite metallic dielectric objects [3]. The MoM problem in [1] was obtained by discretizing with triangular basis functions and this formulation has been adopted in most subsequent works, including [3]. This low-order discretization typically requires 15 unknowns per wavelength to achieve accurate results. Recently, we presented an efficient higher-order BoR-MoM formulation [4], [5] in which the unknown surface current density on the generatrix of the scatterer is discretized using hierarchical Legendre basis functions [6]. Moreover, the generatrix is represented by up to 4th order curvilinear segments that approximate curved surfaces with high accuracy. This combination of a higher-order polynomial current approximation with the smooth geometry implies that the number of unknowns is reduced to approximately 4 per wavelength, i.e., approximately a four-fold reduction compared to the low-order case.

The higher-order BoR-MoM formulation of [4], [5] was used to analyze compact reflector terminals fed by a fundamental-mode circular waveguide and hence, only one azimuthal mode was considered. However, in applications involving non-circular waveguide feeds and/or feeds with offset placement with respect to the axis of symmetry, several azimuthal modes are necessary to accurately describe the incident field.

In this paper we present the higher-order BoR-MoM formulation for arbitrary number of azimuthal modes. Also, to reduce the matrix fill time, the involved modal Green's functions are evaluated asymptotically using the accurate procedure by Gustafsson [7]. Finally, an application example involving an electrically huge dual reflector antenna with diameter 1520 wavelengths and offset feed illumination is considered.

II. The Higher-Order BoR-MoM Formulation

In our BoR-MoM formulation we employ the same continuous integral equation as in previous works, e.g., that of [3], but the equation is discretized with higher-order basis functions of arbitrary order and curved segments of up to 4th order. The basis functions applied here are those of [6], which have been adapted to the present case with rotational symmetry. The electric and magnetic surface current densities, $\vec{J}_s$ and $\vec{M}_s$, on each curve segment are expanded as

$$\vec{X} = \sum_{m=0}^{M^\phi} \vec{X}_m, \quad \vec{X} = \vec{J}_s, \vec{M}_s, \tag{1}$$

$$\vec{X}_m = \sum_{n=0}^{N^t} a_{mn}^{t,e} \vec{B}_{mn}^{t,e} + a_{mn}^{t,o} \vec{B}_{mn}^{t,o} + \sum_{n=0}^{N^t-1} a_{mn}^{\phi,e} \vec{B}_{mn}^{\phi,e} + a_{mn}^{\phi,o} \vec{B}_{mn}^{\phi,o}. \tag{2}$$

Herein, $a_{mn}^{t,e}$, $a_{mn}^{t,o}$, $a_{mn}^{\phi,e}$, and $a_{mn}^{\phi,o}$ are unknown coefficients, N^t is the polynomial expansion order along the generatrix, M^ϕ is the highest azimuthal mode index, and $\vec{B}_{mn}^{t,e}$, $\vec{B}_{mn}^{t,o}$, $\vec{B}_{mn}^{\phi,e}$, and $\vec{B}_{mn}^{\phi,o}$ are t- and ϕ-directed vector basis function, defined as

$$\vec{B}_{mn}^{t,\binom{e}{o}}(t,\phi) = \frac{\vec{a}_t}{\mathcal{J}_s(t,\phi)} \widetilde{P}_n(t) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}, \tag{3a}$$

$$\vec{B}_{mn}^{\phi,\binom{e}{o}}(t,\phi) = \frac{\vec{a}_\phi}{\mathcal{J}_s(t,\phi)} P_n(t) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}. \tag{3b}$$

calculated directivity is shown in Figure 2. The BoR-MoM formulation requires 7300 unknowns. If traditional 3D-MoM with RWG basis functions were applied, approximately 239 million unknowns would be needed. When the integrations of the involved modal Green's functions are performed using trapezoidal quadrature, the total matrix fill time is 20406 s using 8 cores of a Xeon E5-2690 processor. When the steepest descent integration from [7] is used, this fill time is reduced to 8645 s.

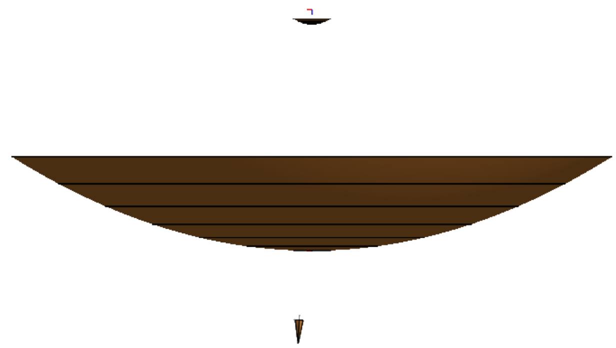

Fig. 1. View of ALMA antenna in GRASP [9]. The main reflector has a centre hole of diameter 0.75 m.

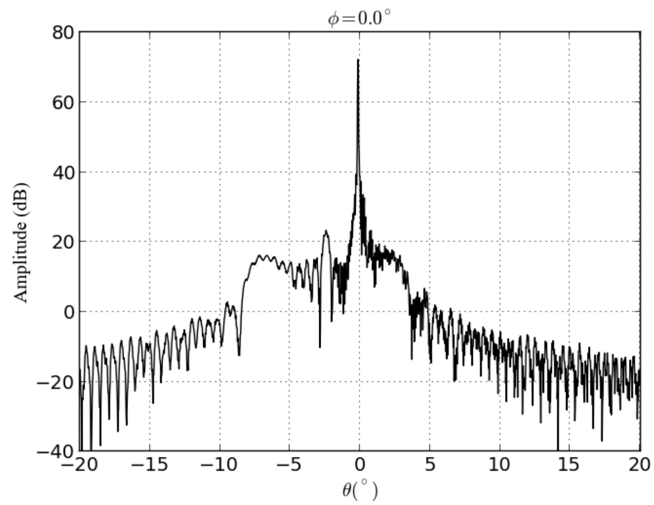

Fig. 2. The directivity in the E-plane as calculated by BoR-MoM.

Here, $\vec{a}_t = \partial\vec{r}/\partial t$, $\vec{a}_\phi = \partial\vec{r}/\partial\phi$, are the covariant unitary vectors, and $\mathcal{J}_s(t,\phi) = |\vec{a}_t \times \vec{a}_\phi|$. In (3b) the polynomials $P_n(t)$ along the direction transverse to the current flow are the Legendre polynomials due to their orthogonality properties. In the direction along the current flow in Eq. (3a), the modified Legendre polynomials

$$\widetilde{P}_n(t) = \begin{cases} 1-t, & n=0 \\ 1+t, & n=1 \\ P_n(t) - P_{n-2}(t), & n \geq 2 \end{cases} \quad (4)$$

are used. The two lowest order polynomials can be matched with the corresponding functions on the neighbouring segments, or alternatively, they can be left out at external nodes.

The expansion order along the generatrix, N^t, is adapted to the electrical length of each segment, which is usually in the order of 2λ. The highest azimuthal mode index, M^ϕ, is determined automatically from the Fourier series expansion of the incident field.

The solutions for different azimuthal modes do not couple and therefore, the MoM problem for the unknowns $a_{mn}^{t,e}$, $a_{mn}^{t,o}$, $a_{mn}^{\phi,e}$, and $a_{mn}^{\phi,o}$ decouples into $M^\phi + 1$ independent smaller MoM problems. Hence, to obtain the solution to the unknown surface current densities, a MoM problem – based on the expansion (2) – is solved for each azimuthal index, and the resulting solutions are summed according to (1).

III. Application Example

To illustrate the efficiency of the BoR-MoM formulation, the Atacama Large Millimeter Array (ALMA) axi-symmetric Cassegrain reflector antenna [8], see Figure 1, is analyzed at the center frequency of Band 1, 38 GHz. The diameter of the main paraboloidal reflector is 12 m, corresponding to 1520 wavelengths, and that of the hyperboloidal subreflector is 0.75 m (95 wavelengths). Also, f/D equals 0.4, the eccentricity of the subreflector is 1.10526, and the distance between the foci is 6.177 m. The subreflector is illuminated by a Gaussian feed with taper -12 dB at 3.6°. The feed is offset 255 mm from the symmetry axis and tilted 2.43°. With this offset, 19 azimuthal modes ($M^\phi = 18$) are needed in the analysis. The

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