Memory-Efficient, Full-Wave Analysis of Waveguide Scattering Parameters for Multiple Reflector Feeds Mounted on Electrically Large Satellite Platforms

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***Abstract*—A Higher-Order Multilevel Fast Multipole Method (HO MLFMM) is presented for the efficient calculation of waveguide scattering parameters in cases where several reflector feeds are mounted on electrically large satellite platforms. The efficiency is obtained by: 1) using a Block Krylov method for solving the HO MLFMM problem for the required large number of right-hand sides and 2) implementing an out-of-core solution, taking into account the fact that the memory distribution of HO MLFMM is different from the standard low-order MLFMM.**

I. Introduction

Multiple antenna systems on satellite platforms are often tightly packed, and the entire scattering matrix is needed for accurate analysis and design. Space applications typically require extreme accuracy, with a desired dynamic range often exceeding 100 dB, and a full-wave method is therefore required to determine the scattering matrix. At the same time, most satellite platforms are electrically large, and a memory-efficient full-wave method is fundamental if analysis should be performed on computers with modest memory.

The Higher-Order Multilevel Fast Multipole Method (HO MLFMM), recently introduced in [1], is capable of reaching a high level of accuracy with a very small memory footprint, and is therefore a perfect candidate for the above-mentioned full-wave method. However, to compute a P by P scattering matrix, the MLFMM problem must be solved for P right hand sides (RHS), resulting in long solution time. Several approaches for MLFMM solutions with multiple RHS have been developed for monostatic RCS applications, but these methods are mainly applicable when there is a strong correlation between the various RHS. To reduce the solution time for scattering matrix computations, we have introduced the block GMRES solver into the HO MLFMM algorithm and show that this solver reduces the solution time.

In addition, the HO approach implies that a relatively large part of the memory is occupied by the near-interaction matrix, whereas the memory required for the MLFMM acceleration occupies a smaller part of the memory. This property means that an Out-of-Core (OoC) HO MLFMM solution, which employs disk storage instead of RAM, is much more memory efficient than the OoC version of the standard, widely used low-order MLFMM implementation. Hence, the OoC HO MLFMM is well-suited for extending the range of solvable problems on a given computer hardware. In this paper we investigate the memory distribution of the HO MLFMM and illustrate the performance of the OoC HO MLFMM.

II. HO MLFMM with Multiple Right-Hand Sides

When computing the scattering matrix of a multiport antenna system, a system of equations with multiple right-hand sides is set up

$$\overline{\overline{Z}}\,\overline{\overline{I}} = \overline{\overline{V}} \tag{1}$$

where $\overline{\overline{V}}$ and $\overline{\overline{I}}$ have N rows and P columns. When N is large, these problems are typically solved by P consecutive applications of an iterative linear solver. In some applications, the convergence for the $(i+1)$'st right-hand side can be improved by using information from the solution of the i'th right-hand side. Unfortunately, this trick cannot be applied when the excitations are uncorrelated. A more thorough approach is the use of Block Krylov solvers - these solve all P systems simultaneously, requiring P matrix-vector products with $\overline{\overline{Z}}$ per iteration, but compressing all the information from those P systems into one Krylov subspace. One such method, the Block GMRES, is described in detail in [2] and this method has been implemented in the HO MLFMM solver.

As a simple example of the properties of the Block GMRES, we consider an offset paraboloidal reflector with circular projected rim, illuminated by an axially corrugated horn designed for use in the 20-30 GHz range. We fix the frequency at 30 GHz, resulting in an overmoded horn and include 20 waveguide modes in the scattering matrix computation to ensure that the higher-order modes are not detrimental to the performance. The scenario is illustrated in Fig. 1, and the total number of RHS in the system is $P = 20$.

The performance of the Block GMRES solver is contrasted with the standard P applications of non-restarted GMRES in Table I, with the relative residual tolerance of 10^{-3}. The key performance parameter is the number of matrix-vector products required for all P right-hand sides to achieve convergence - for the standard GMRES this is 354 while it is 280 for Block GMRES. Thus, the Block GMRES reduces the total number of matrix-vector products with about 20% for this specific case.

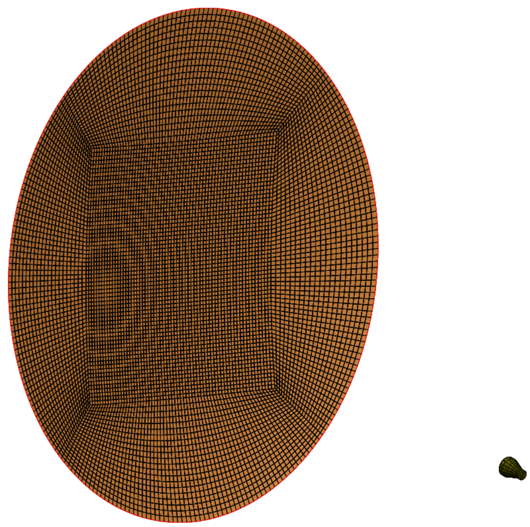

Fig. 1. Mesh of the offset $D = 100\lambda$ paraboloidal reflector, illuminated by an axially corrugated horn, used as a test case for the Block GMRES solver. Note that each patch has roughly 1.5λ sidelength.

TABLE I
NUMBER OF MATRIX-VECTOR MULTIPLIES

Method	Number of runs	Number of iterations	Matrix-vector products per iteration	Matrix-vector products Total
GMRES	20	13-22	1	354
Block GMRES	1	14	20	280

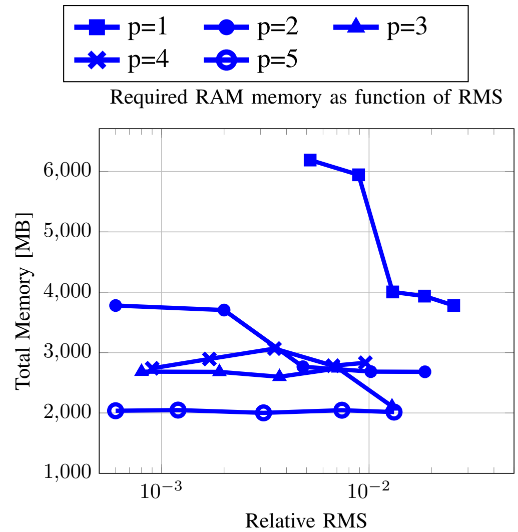

Fig. 2. Memory (RAM) required by the OoC HO MLFMM solver for various expansion orders.

TABLE II
OUT-OF-CORE PERFORMANCE ON A PARABOLOIDAL REFLECTOR WITH DIAMETER D.

D $[\lambda]$	In-core Memory [GB]	In-core Time	Out-of-Core Memory [GB]	Out-of-Core Time
50	0.68	1:11 min	0.11	1:37 min
100	2.60	5:05 min	0.32	6:43 min
200	10.25	24:11 min	1.16	40:00 min
400	20.08	**N/A**	4.42	4:11 hrs

III. OoC HO MLFMM Solver

Although the HO MLFMM algorithm is extremely memory efficient, there is still a strong demand for solving problems requiring more memory than available. This need can be addressed by developing an OoC solver, thereby reducing the peak memory requirement at the expense of a longer solution time.

For low-order MLFMM, approximately 50% of the memory is used for the near matrix and the basis functions patterns. The remaining part of the memory is occupied by group patterns, translation operators, and other MLFMM data needed several times in each iteration. For HO MLFMM, however, approximately 75% of the memory is used for the near matrix and the basis functions patterns. Since a larger part of the memory is occupied by data that is only needed once per iteration, the HO MLFMM is more suitable for an OoC solution than low-order MLFMM.

The difference outlined above and the polynomial expansion order have a direct impact on the amount of data that can be stored on disk and the data that must be kept in RAM. This is illustrated in Figure 2, showing the amount of RAM needed when the OoC HO MLFMM is used to solve the same test case as studied in the previous section. It is observed that the memory consumption decreases significantly as the expansion order increases. Also, the curve for the highest expansion order $p = 5$ is almost flat, indicating that the memory requirement is independent of the solution accuracy.

The performance of the OoC HO MLFMM algorithm is now illustrated by considering the paraboloidal reflector of Figure 1 with different diameters D between 50λ and 400λ. The radiation patterns have been computed with HO MLFMM on a laptop with 16 GB RAM. The memory and CPU time are reported in Table II for both the in-core and the out-of-core solution. It is observed that the required RAM is between 5 and 10 times lower with the OoC solver, at the expense of a longer runtime.

IV. Conclusion

We have shown that a Block GMRES solver can be used to reduce the iteration time when the HO MLFMM is applied for computation of scattering matrices.

We have also shown that the HO MLFMM results in a relatively large near matrix which makes the algorithm suitable for an out-of-core solution. When the OoC HO MLFMM is used, a very high solution accuracy can be obtained by p-refinement without affecting the required in-core memory.

At the conference we will illustrate the performance of the Block GMRES solver and OoC HO MLFMM for cases involving electrically large satellite platforms.

References

[1] O. Borries, P. Meincke, E. Jørgensen, and P. C. Hansen, "Multilevel fast multipole method for higher order discretizations," *IEEE Trans. Antennas Propagat.*, vol. 62, no. 9, pp. 4695 – 4705, Sep. 2014.

[2] M. H. Gutknecht, "Block Krylov space methods for linear systems with multiple right-hand sides: an introduction," *Seminar for Applied Mathematics*, 2006.