Higher-Order Method of Moments Analysis for Unconnected Quadrilateral Meshes

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***Abstract*—A higher-Order (HO) quadrilateral mesher is presented along with a HO method of moments formulation for unconnected meshes. A numerical example is presented to validate the new formulation.**

I. INTRODUCTION

Higher-Order (HO) Method of Moments (MoM) [1] relies on a HO polynomial expansion of the unknown surface current density on large (up to 2 by 2 wavelengths), HO (curved) quadrilateral patches. In comparison to the widely used low-order approach based on RWG basis functions [2], i.e., first-order approximations in both current expansion and geometry, the HO formulation is much more efficient in terms of memory consumption and computational speed. However, higher efficiency is only obtained if the quadrilateral patches are as large as possible throughout the entire mesh. For large surfaces containing small geometrical structures, for instance holes or rapidly varying rims, this requirement constitutes a challenge to the meshing procedure: Small patches should only be generated near the small geometrical details, and not throughout the entire surface. Moreover, for complicated structures obtained from CAD, typically containing numerous surfaces, these surfaces are usually meshed independently with the requirement that the mesh is connected at edges joining two surfaces. The connectivity requirement – which is a result of the continuous current assumption of the mixed-potential MoM-formulation – implies that small patches on one surface, as a result of small geometry details, also force small patches on the connected surfaces, thus jeopardizing the HO-MoM efficiency.

In this paper we present a remedy to the two problems presented above: local geometrical details and mesh connectivity. First, in Section II, a HO quadrilateral mesher is discussed, with the capability of generating meshes of varying patch size on a single surface. Second, in Section III, we present a Discontinuous Galerkin Integral Equation (DGIE) formulation for HO-MoM that allows an unconnected HO quadrilateral mesh, thus eliminating the connectivity requirement across edges of two joined surfaces. Finally, a numerical example is presented in Section IV, showing that the DGIE-formulation for an unconnected HO mesh yields the same results as does the usual MoM-formulation for a connected HO mesh.

II. HIGHER-ORDER QUADRILATERAL MESHING

Two main approaches exist for producing quadrilateral meshes: indirect and direct methods. With indirect methods a triangular mesh is first made, and this mesh is subsequently transformed into the desired quadrilateral one. With direct methods the quadrilaterals are generated directly on the surface. *Paving* by Blacker and Stephenson [3], which is a direct method, appears to be the most promising approach, particularly because it allows for high mesh quality also when patches of varying sizes are present on a surface due to different geometrical details. Paving, which was originally developed for 2D-surfaces, is an advancing front method where complete rows of elements are inserted, starting from the exterior boundary and proceeding towards the interior of the surface. If the surface has holes, paving is also performed from these interior boundaries, working towards the outer part of the surface. The paving algorithm has been extended to 3D-surfaces [4], to element- instead of row-based insertion [5], and optimized for electromagnetic tools [6]. The meshes shown in all these papers have in common that the seed points on the boundaries are equidistant, resulting in patches of equal size on the entire surface. Hence, if small geometrical details are present, the mesh size is reduced accordingly on the entire structure, which is not efficient for HO-MoM.

In this work we have optimized the paving approach to work with seed points of non-equidistant spacing. With this implementation, the patch size can easily be adjusted across a surface with varying levels of geometrical details, an example of which is shown in Figure 1.

III. METHOD OF MOMENTS FORMULATION FOR UNCONNECTED MESHES

In the widely used mixed-potential MoM formulation it is required that the normal component of the current is continuous across patches, and this forces the mesh to be connected. Figure 2 illustrates a problem associated with this fact. The attachment of the boom to the upper surface close to the edge implies small patches in this region, and due to the connectivity requirements, small patches are also present in the mesh of the side surface.

Peng *et al.* [7] have recently developed a DGIE-formulation that eliminates the continuity requirement of the current, thus allowing unconnected meshes. The formulation was demonstrated for RWG basis functions and flat, triangular meshes. We have modifed the DGIE formulation to work for HO basis functions and HO quadrilateral meshes. With this new formulation, the unconnected mesh of Figure 3 can be used in the analysis. In this mesh, as opposed to the connected

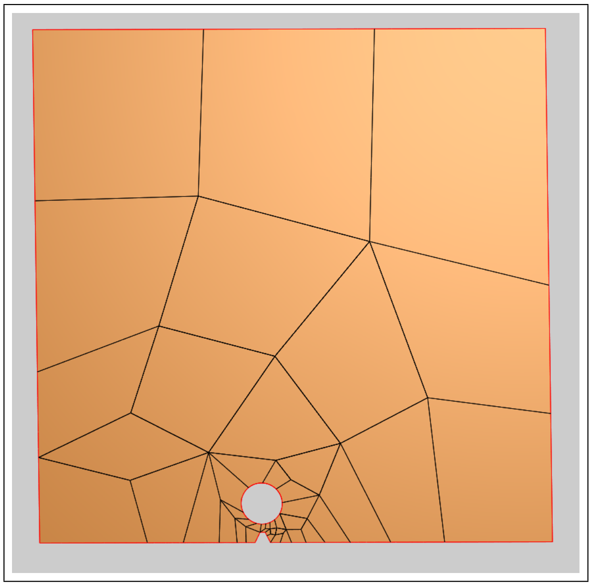

Fig. 1. Example illustrating the capability of the implemented paving mesher to adapt to geometrical details of varying size.

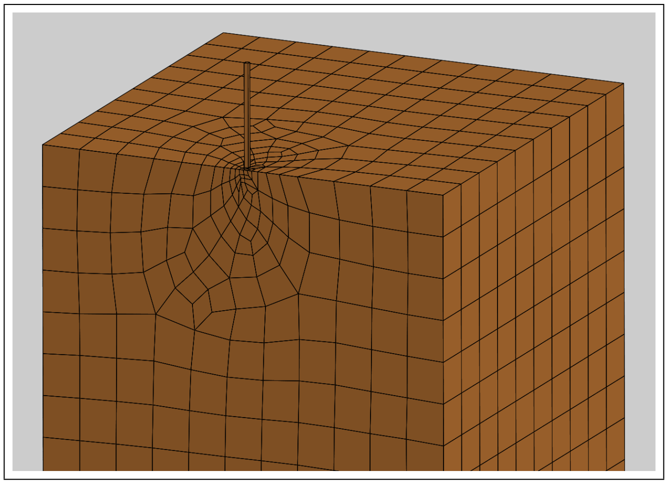

Fig. 2. Connected mesh for standard MoM.

counterpart in Figure 2, the mesh on the side is not affected by the presence of the small patches near the boom.

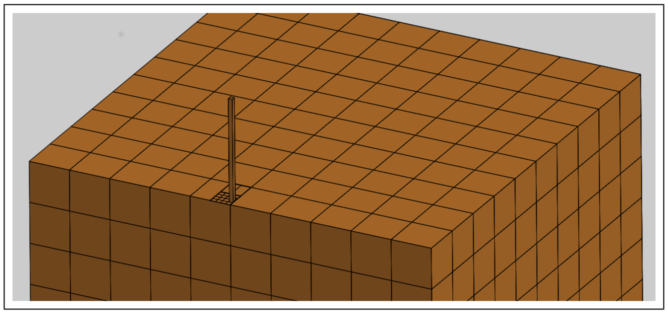

Fig. 3. Unconnected mesh for DGIE formulation.

IV. EXAMPLE

The paving mesher and the DGIE formulation of Sections II and III have been implemented in TICRA's GRASP software [8], and subsequently used to calculate the surface current density on a 10λ by 10λ by 10λ box illuminated by a Gaussian feed, located and oriented to create a strong excitation of two of the faces and their common edge. The problem is analyzed using three different meshes, two connected and one unconnected. As seen in the same figure, the calculated surface current density is identical for the three meshes, thus validating the DGIE formulation. See [9] for more advanced examples.

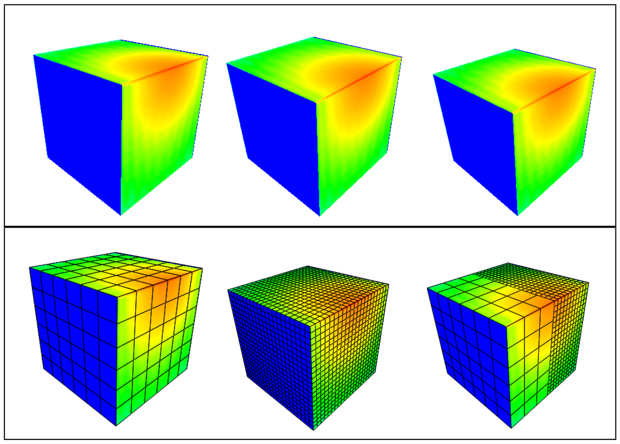

Fig. 4. Induced surface current density obtained on three different meshes. Left column: coarse mesh, centre column: fine mesh, right column: unconnected mesh with DGIE. The bottom row shows the same currents as the top row, but mesh lines have been included to better illustrate where potential current artifacts could be located.

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