Efficient Optimization of Large Reflectarrays Using Continuous Functions

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***Abstract*—Two new efficient optimization schemes for direct optimization of printed reflectarrays are presented. Instead of optimizing directly on the geometry of the array elements, continuous function are envoked to represent the aperture field from which the geometry of the array elements are determined. In this way, the sharp transitions in the geometrical variation of the array elements can be sufficiently reproduced and gives designs that are comparable to those obtained where the array elements are directly optimized. The first scheme uses complex spline representation (CSR) whereas the second uses Fourier series representation (FSR). To demonstrate the two schemes, an offset 50×50 square wavelengths pencil beam reflectarray is considered. It is shown that the CSR and FSR schemes are capable of yielding designs with the same performance as that obtained where the array elements are directly optimized, and that the optimization time can be reduced by a factor of 6.**

I. Introduction

Printed reflectarrays have many advantages compared to conventional reflector antennas. They are flat, light, easy and cheap to manufacture, and provide a way to realize low-cost high-gain antennas for space applications and have therefore been the subject of increasing research and development activities [1], [2].

To ensure high gain, the electrical size of printed reflectarrays must be large and an accurate and efficient design procedure is a challenging task. The conventional approach for the design of printed reflectarrays is based on a phase-only optimization technique [3]–[5] involving two steps. Initially, a phase-only synthesis is performed to determine the phase distributions on the reflectarray surface. The array elements are subsequently adjusted, element by element, to match the required phase distributions obtained in the first step.

Although the phase-only approach is efficient, it suffers the drawback that intermediate optimization steps are necessary to fulfill a given phase distribution. A direct optimization technique, where all the array elements are simultaneously optimized, may produce more optimal designs. Such a direct optimization technique was presented in [6], where several contoured beam reflectarrays were designed and compared to similar designs obtained using the phase-only optimization technique. The comparisons showed that the designs obtained using the direct optimization technique are superior in performance, both for multi-frequency and dual-polarization designs.

For large reflectarrays, where the number of array elements is large, the number of optimization variables becomes prohibitively large and the optimization can be slow. Compared to commercially available software packages for the design of shaped reflectors, e.g. POS [7], the overall optimization time is still high. The objective of this paper is therefore to reduce the optimization time, by using new optimization schemes that use continuous functions in the optimization to determine the geometry of the array elements. Only reflectarrays consisting of square patches of varying sizes are considered in this paper.

This paper is organized as followed. The analysis and optimization methods used in the direct optimization technique are reviewed in Section II. Section III describes the new optimization schemes and representative results are presented in Section IV. Conclusions are given in Section V.

All the computations reported in this paper are carried out on a $2.6\,\mathrm{GHz}$ quad-core Intel processor laptop computer.

II. Analysis and Optimization

The analysis is based on the spectral domain method of moments assuming local periodicity (LP-SDMoM) [8]. For the efficient and accurate calculation of the electric currents on the array elements, higher-order hierarchical Legendre basis functions [9] are used in the LP-SDMoM. It was demonstrated in [10] that these basis functions are capable of providing accurate results and at the same time maintain a low computation time compared to other types of basis functions. The flexibility of the higher-order hierarchical Legendre basis functions allows the optimization of reflectarrays consisting of non-canonical elements shapes, which is an important feature in reflectarray analysis and optimization.

The optimization uses a gradient-based method for non-linear minimax optimization. It is the same optimization algorithm used in the TICRA software packages POS and CHAMP [7].

The far-field requirements are specified at a number of far-field points and the geometrical parameters of the array elements, e.g. the size and orientation of the array element, are optimized at each iteration to minimize the maximum difference between realized and specified objectives. Both co- and cross-polar radiation can be optimized for multiple frequencies, polarizations, and feed illuminations.

During the optimization, the far-field is calculated using the Floquet harmonics technique [11, Technique II]. The technique is based on the field equivalence principle and utilizes scattering matrices which are calculated through the fundamental Floquet harmonics. For the evaluation of the final optimized reflectarray, a more refined technique, the continuous spectrum

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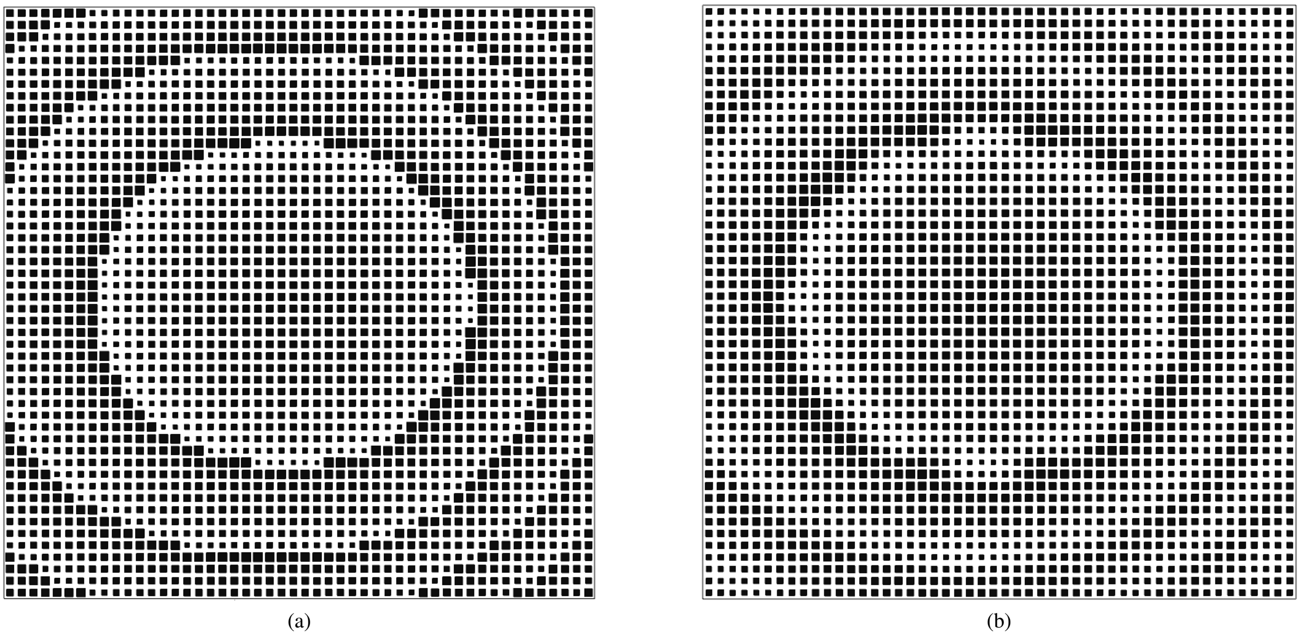


Fig. 1. The mask layout of pencil beam reflectarrays (a) where the array elements are directly optimized and (b) where spline representation is used to represent the geometry of the array elements.

technique [11, Technique III], is used. It is also based on the field equivalence principle, but uses a continuous spectrum to calculate the equivalent currents.

To avoid the calculation of the scattering matrices of all the array elements at each iteration, the scattering matrices are determined prior to the optimization and stored in a look-up table, which can be accessed during the optimization.

For more details on the analysis and optimization, the reader is referred to [6], [10]–[12]

III. CSR and FSR Optimization Schemes

One way to reduce the overall optimization time is to decrease the number of optimization variables N_v. This can be achieved by e.g. representing the geometrical variation of the array elements over the reflectarray surface using continuous functions, e.g. cubic splines

$$s(x,y) = \sum_{m=1}^{M}\sum_{n=1}^{N} c_{mn} B_m(x) B_n(y). \tag{1}$$

Herein, $s(x,y)$ describes the geometry of the array elements at coordinate (x,y), c_{mn} are the spline coefficients, and $B_m(x)$ and $B_n(y)$ are cubic B-splines. The spline coefficients c_{mn} are the optimization variables used to optimize the geometry of the array elements. The idea is to use relatively few spline coefficients to adequately represent the required geometrical variation.

For reflectarrays consisting of array elements of varying sizes, the variation of the size of the array elements over the reflectarray surface can have discontinuities where the scattered phase is required to jump after a complete 360° cycle. Such discontinuities are hard to represent using splines as is shown in Fig. 1. In this figure, the mask layout of two pencil beam reflectarrays are shown, one obtained where the array elements are directly optimized (Fig. 1a), and one obtained where splines are used to represent the geometry of the array elements (Fig. 1b). It is seen for the spline design, with 30×30 splines, that the sharp transition in the variation of the patch sizes is not well represented.

To improve the spline representation, a periodic mapping between $s(x,y)$ and the geometry of the array elements was implemented such that the discontinuities are better accounted for. However, even in this case, the mask layout of the reflectarray is too complicated to be represented by means of splines. As a consequence, the performance of a design obtained using splines, or other continuous functions, to represent the geometry of the array elements is inferior compared to that of a design where the array elements are directly optimized.

In the attempt to alleviate this issue, two new optimization schemes have been implemented. The idea is to use continuous functions to represent field quantities rather than the geometry of the array elements. The two schemes will be described in the following.

A. Complex Spline Representation

Suppose that the dominant component of the aperture field over the reflectarray surface can be expressed by

$$\alpha(x,y) = A(x,y)e^{j\Phi(x,y)}, \tag{2}$$

where $A(x,y)$ and $\Phi(x,y)$ describe the amplitude and phase distribution, respectively.

In this optimization scheme, which will be denoted the complex spline representation (CSR), cubic splines are not employed to represent $s(x,y)$ as in (1), but rather to reproduce the aperture field

$$\alpha(x,y) = \sum_{m=1}^{M}\sum_{n=1}^{N}(a_{mn} + jb_{mn})B_m(x)B_n(y). \quad (3)$$

Herein, the spline coefficients a_{mn} and b_{mn} are the optimization variables. From $\alpha(x,y)$, the phase distribution $\Phi(x,y)$ can be found as

$$\Phi(x,y) = \mathrm{Arg}\big(\alpha(x,y)\big) = \mathcal{I}m\big[\ln\big(\alpha(x,y)\big)\big], \quad (4)$$

where Arg denotes the principle value of the argument. In reflectarray designs, the array elements constitute the phase-shifters used to adjust the phase distribution, thus there is a direct relation between $\Phi(x,y)$ and $s(x,y)$. In the CSR scheme, we assume a linear relation between $\Phi(x,y)$ and $s(x,y)$ such that $\Phi(x,y) = -\pi$ equals the minimum element size and $\Phi(x,y) = \pi$ the maximum element size. Thus, by optimizing a_{mn} and b_{mn}, the geometry of the array elements is obtained.

The derivatives wrt. the spline coefficients, which are required in the optimization, can be calculated analytically by differentiation of the logarithm function in (4).

The drawback of this scheme is that complex optimization variables are needed, giving twice the optimization variables as compared to (1). However, this scheme is more capable of representing the sharp transitions in the variation of the array elements as will be demonstrated later.

B. Fourier Series Representation

This optimization scheme is similar to CSR. However, instead of using cubic splines, a Fourier series is utilized to represent $\Phi(x,y)$, and hence the name Fourier series representation (FSR),

$$\alpha(x,y) = \sum_{m=-M}^{M}\sum_{n=-N}^{N}(a_{mn} + jb_{mn})e^{j2\pi\left(\frac{mx}{d_x}+\frac{ny}{d_y}\right)}. \quad (5)$$

The quantities d_x and d_y are the x and y dimensions of the reflectarray, see Fig. 2, and the Fourier coefficients a_{mn} and b_{mn} are the optimization variables used to determine the geometry of the array elements in the same way as in the CSR scheme.

IV. Numerical Results

To demonstrate the different optimization schemes, we consider an offset $1.5\times1.5\,\mathrm{m}^2$ pencil beam reflectarray optimized to operate in the frequency range $9.5-10.5\,\mathrm{GHz}$. At the centre frequency $10\,\mathrm{GHz}$, the dimension of the reflectarray corresponds to $50\times50\,\lambda_0^2$, where λ_0 is the free-space wavelength. The geometrical parameters are listed in Table I with respect to the coordinate system shown in Fig. 2. As feed, a linearly polarized Gaussian beam with a taper of $-25\,\mathrm{dB}$ at $30°$ is used.

To serve as reference solution, a design where the size of the array elements are directly optimized has been obtained. For this design, the number of optimization variables is $N_\mathrm{v} =$

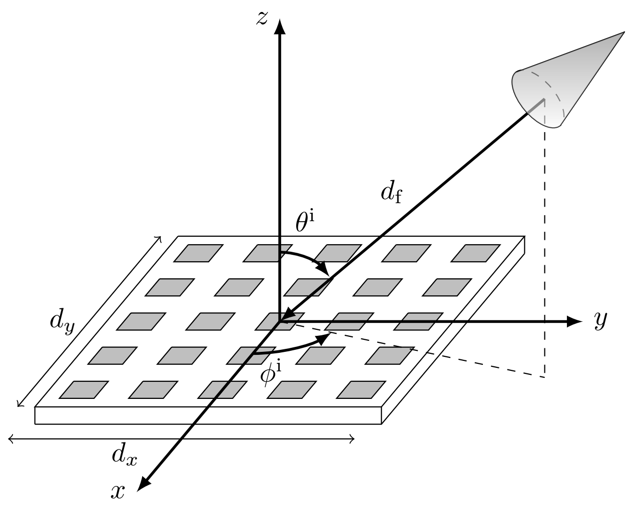


Fig. 2. Reflectarray geometrical parameters.

TABLE I
Reflectarray Data

Frequency	$9.5-10.5\,\mathrm{GHz}$
Reflectarray dimension	$1.5\,\mathrm{m}\times1.5\,\mathrm{m}$
Number of elements	110×110
Relative permittivity	$\epsilon_\mathrm{r} = 3.66$
Loss tangent	$\tan\delta = 0.0037$
Substrate thickness	$h = 1.524\,\mathrm{mm}$
Feed distance	$d_\mathrm{f} = 2.0\,\mathrm{m}$
Feed offset angle	$\theta^\mathrm{i} = 30°$, $\phi^\mathrm{i} = 0°$
Main beam direction	$\theta = -30°$, $\phi = 0°$

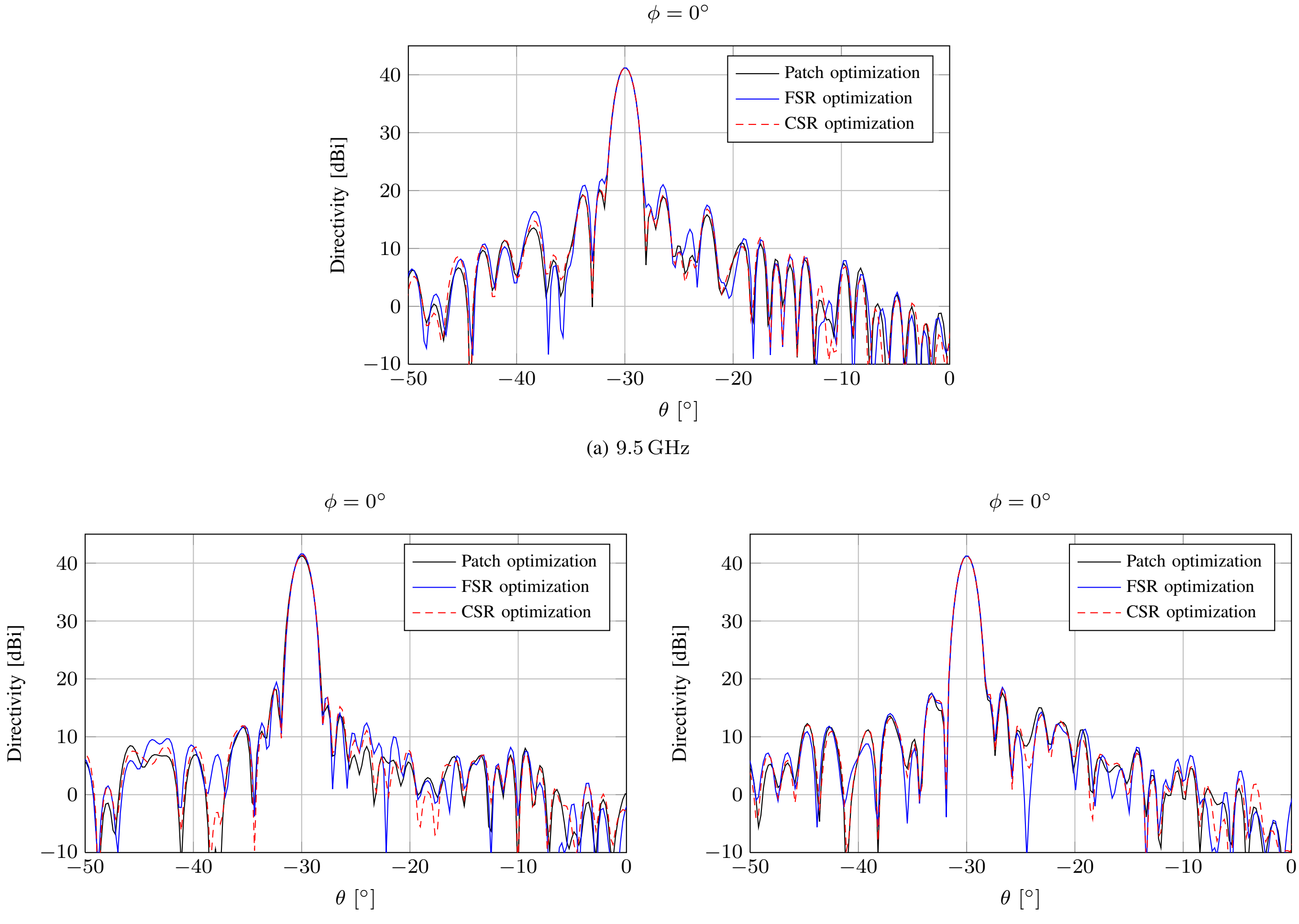
TABLE II
Performance of Reflectarray Designs

Patch				FSR		CSR	
N_v	D_0 [dBi]	t [sec.]	N_v	D_0 [dBi]	t [sec.]	D_0 [dBi]	t [sec.]
			3362	41.0	46	40.5	11
			5202	41.2	74	40.9	28
12100	41.2	374	7442	41.2	90	41.1	42
			10082	-	-	41.2	62
			13122	-	-	41.2	186

12100, which is equal to the number of array elements, and the overall optimization time was $t = 374\,\mathrm{s}$. The minimum peak directivity for this design in the frequency range $9.5-10.5\,\mathrm{GHz}$ is $D_0 = 41.2\,\mathrm{dBi}$.

The FSR and CSR optimization strategies were then used to obtain similar designs and the results are summarized in Table II. Herein, the number of optimization variables N_v and optimization time t together with the obtained D_0 are listed.

For the FSR optimization, it was observed that $D_0 = 41.2\,\mathrm{dBi}$ can be achieved by selecting $M = N = 25$, resulting in $N_\mathrm{v} = 5202$ and an optimization time of $t = 74\,\mathrm{s}$. For the CSR optimization, the same performance can be obtained using $M = N = 71$, which gives $N_\mathrm{v} = 10082$ and an optimization time of $t = 62\,\mathrm{s}$. For both FSR and CSR, the optimization time is lower than that of the direct array element optimization, where t can be reduced by a factor of approximately 6.



(a) 9.5 GHz

(b) 10 GHz

(c) 10.5 GHz

Fig. 3. The radiation patterns of pencil beam reflectarray designs obtained by directly optimizing the array elements (black), by using the FSR (blue) and CSR (red) optimization schemes.

The radiation pattern of these designs for $f = 9.5\,\text{GHz}, 10\,\text{GHz}, 10.5\,\text{GHz}$ are shown in Fig. 3. It is seen that the radiation properties of the CSR and FSR designs are very similar to that obtained using the direct array element optimization. The mask layout of the CSR design is shown in Fig. 4, and it is apparent that the sharp transitions in the variation of the patch sizes are well represented. A similar observation was seen for the FSR design.

It is seen in Table II that the optimization time for CSR is lower than that of the FSR, even though N_v are the same. This is due to the overhead involved in the FSR when calculating the derivatives. This can be improved if an FFT algorithm is used to evaluate (5).

It is also noted that t for the CSR design using $N_\text{v} = 13122$ is lower than that of the direct array element optimization, even though N_v is higher. This is explained by the faster convergence of the CSR optimization. The number of optimization iterations N_iter required to obtain the final design for the CSR, and also the FSR, is usually between $50 - 100$, whereas it is around $N_\text{iter} = 200$ for the direct patch optimization.

For the pencil beam reflectarrays considered in this paper,

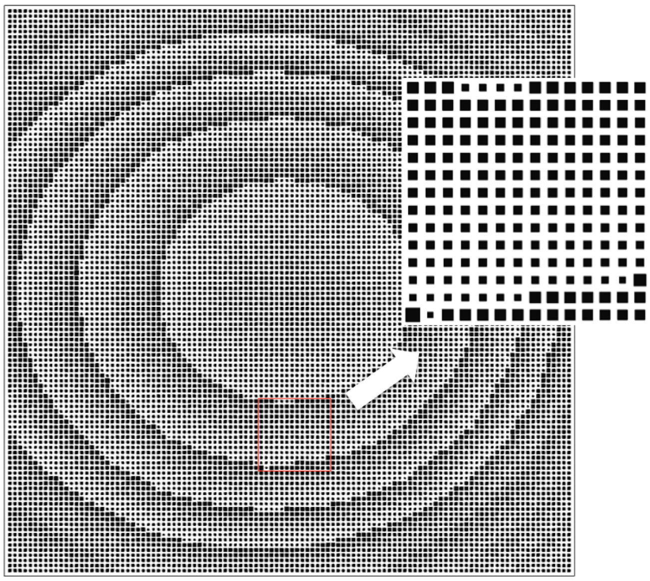


Fig. 4. Mask layout of design obtained using the CSR optimization scheme.

the FSR and CSR optimization schemes are fairly efficient where a factor of 6 can be gained in terms of the optimization time. However, for more complex designs e.g. contoured beam and multi-beam reflectarrays, where the mask layout are more complicated, no significant advantage is obtained using FSR and CSR. Thus, improvements are still needed and techniques to enhance the optimization schemes are currently being investigated.

V. Conclusion

Two new efficient optimization schemes for the direct optimization of large printed reflectarrays have been presented. Instead of optimizing directly on the geometry of the array elements, continuous functions are utilized in the optimization to represent the aperture field from which the geometry of the array elements are determined. The first optimization scheme uses complex spline representation (CSR) whereas the second uses Fourier series representation (FSR). For both schemes, the sharp transitions in the variation of the array elements can be adequately reproduced and yields designs that are similar to that obtained where the array elements are directly optimized. This cannot be achieved if the continuous functions are used to directly represent the geometry of the array elements.

To demonstrate the optimization schemes, an offset $1.5 \times 1.5\,\mathrm{m}^2$ pencil beam reflectarray optimized to operate in the frequency range $9.5-10.5\,\mathrm{GHz}$ was considered. It was shown that the CSR and FSR optimization schemes are capable of yielding designs with the same performance as that obtained where the array elements are directly optimized, and that the optimization time can be reduced by a factor of 6. For complicated designs such as contoured beam reflectarrays, enhancements are still required and techniques to improve the optimization is currently being investigated.

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