1 Introduction

The effects of reflector surface distortions for a paraboloidal pencil beam reflector antenna are well understood. The peak gain reduction is proportional to the surface error squared, whereas the influence in the side-lobe region is linearly proportional to the surface error and the angular distribution depends on the correlation distance (the surface roughness).

For shaped reflectors the effects become more complicated. It turns out that the gain reduction in the coverage region is proportional to the surface error and leading to significantly increased surface tolerance requirements.

This paper describes the influence of surface errors for three reflectors shaped for uniform circular coverage with half angles of $2^\circ$, $4^\circ$ and $8^\circ$. For the $4^\circ$ case an equivalent multibeam antenna is investigated for comparison.

2 Design of three single feed, shaped beam antennas

As an initial configuration is selected a rotationally symmetric parabolic front-fed reflector antenna with focal length $f = 50\lambda$, diameter $D = 50\lambda$, $\lambda = \text{wavelength}$ and the feed taper is 18 dB. This antenna will generate a narrow beam with a 3 dB beamwidth of $1.47^\circ$.

The reflector surface shape is now optimized to generate a uniform far field with the maximum possible minimum gain within a circular cone of half-angle $\theta_b$, $\theta_b = 2^\circ$, $4^\circ$ and $8^\circ$. The reflector surface is represented by a series of Zernike modes and due to the rotational symmetry only $m = 0$ modes need be included in the optimization. These modes are polynomials of even order in the radial distance $r$ and the lowest order term is a paraboloid with focal length $f_c$. During the optimization the field is calculated by Physical Optics (PO). The resulting surface shapes are illustrated in Figure 1 where the dominant paraboloidal term is shown in true scale but the sum of all the higher order terms is multiplied by 20 for better visibility.

The main characteristics of the optimized reflectors are summarized in Table 1. $E_{GO}$ is the Geometrical Optics (GO) limit when all power is uniformly distributed in the coverage area. $E_{c}$ is the realized minimum coverage gain. $N_{\text{max}}$ is the number of modes used in the surface representation and $f_c$ is the focal length of the first term, i.e. the best-fit paraboloid. For each far-field direction, $\theta$, in the coverage region the reflector aperture phase pattern will contain a stationary point around which the iso-phase contours will be approximately circular. The reflection spot size, denoted $a$ in the right column in Table 1, is here defined as the diameter of the phase contour which is $90^\circ$ out of phase with respect to the stationary point. The reflection spot size will depend on the far-field direction and the value given in Table 1 is for $\theta = \theta_b/2$. The reflection spot size represents the active area of the reflector and it becomes an important parameter when introducing surface errors in the following section. Under GO approximations one finds that the reflection spot size is inversely proportional to $\theta_b$ and the values in Table 1 agree reasonably well with this rule.

The calculated patterns for the optimized reflectors are shown in Figure 2.

![Figure 1 Reflector surfaces shaped for $2^\circ$, $4^\circ$ and $8^\circ$ coverage, $D/\lambda=50$](image-url)
3 Performance degradation due to surface errors

For a general investigation of surface distortions it is convenient to be able to generate such errors in a systematic manner. This can be done by superimposing a square grid on the undistorted reflector surface, as shown in Figure 3. The node values are selected as random numbers uniformly distributed in a given interval, ±δ, and with a mean value equal to zero. A cubic interpolation function yields a smooth surface between the random values at the nodes. The spacing between the nodes, c, relative to the reflector diameter, D, determines the roughness of the surface. Figure 4 illustrates the model for c/D = 0.1. In this way the correlation distance is 2c, meaning that within a circular area of diameter 2c the surface distortions are correlated, whereas they are nearly uncorrelated for larger distances. The rms surface error can be shown to be $e_{\text{rms}} = 0.47\delta$.

It is reasonable to assume that the node spacing $c$ relative to the reflection spot size is an important parameter for the influence of the surface distortion. We have therefore investigated $c = 2\delta$, $\delta/2$, $\delta/4$, $\delta/8$ and $\delta/16$ for each of the three reflector systems, where $\delta$ is given in Table 1. The distortion amplitude is selected to $\delta = \lambda/40$ and $\lambda/80$. These values correspond to $e_{\text{rms}} = 0.012\delta$ and $0.006\delta$, respectively, and they would - for a focusing paraboloidal reflector - result in a peak gain loss of only 0.1 and 0.02 dB, respectively.

<table>
<thead>
<tr>
<th>Coverage region half angle, $\theta_b$</th>
<th>Ideal upper limit, $E_{\text{GO}}$ dBi</th>
<th>Realized gain $E_o$ dBi</th>
<th>Number of modes $N_{\text{max}}$</th>
<th>bfp focal length, $f_c/\lambda$</th>
<th>Reflection spot size, $a/\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>$\infty$</td>
<td>42.52</td>
<td>1</td>
<td>50.00</td>
<td>-</td>
</tr>
<tr>
<td>$2^\circ$</td>
<td>35.16</td>
<td>32.25</td>
<td>5</td>
<td>54.00</td>
<td>22.97</td>
</tr>
<tr>
<td>$4^\circ$</td>
<td>29.14</td>
<td>27.59</td>
<td>7</td>
<td>62.55</td>
<td>16.50</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>23.13</td>
<td>21.73</td>
<td>11</td>
<td>90.33</td>
<td>13.26</td>
</tr>
</tbody>
</table>

Table 1 Characteristics of shaped reflector systems, D/\lambda=50.

Figure 2 Shaped beams for 2°, 4° and 8° coverage and the unshaped pencil beam, $\theta_b = 0^\circ$. D/\lambda=50.
Figure 3  Grid for reflector distortion definition

Figure 4  Surface error model for $c/D = 0.1$
For each combination of coverage size $\theta_b$, node spacing $c$ and distortion amplitude $\delta$ the coverage field is calculated for 40 different seeds to the random number generator and the minimum value is identified. Finally, the average value of the 40 minimum values is calculated and the result is plotted in Figure 5. It must be pointed out that for each distortion case only one station in the coverage suffers from the maximum gain loss and for many other stations the gain will actually have increased due to the surface distortions. However, standard antenna specifications refer to all stations within the coverage area so in general it is not possible to take advantage of this fact.

The results in Figure 5 clearly show that the gain loss is highly dependent upon the correlation distance of the surface distortions. The maximum appears when the node spacing is about half the reflection spot size and it is seen that the gain loss is more than 2 dB for the broadest beam, $\theta_b = 8^\circ$, for a distortion amplitude of $\delta = \lambda/40$. It may be shown theoretically that when the node spacing is much larger than the reflection spot size, $c \gg a$, the gain loss will decrease as $1/c^2$, and when the node spacing is much smaller than the reflection spot size, $c \ll a$, the gain loss will be proportional to $c$.

The numerical results in Figure 5 confirm these rules perfectly.

Figure 5 also reveals that a broad beam is more sensitive to surface errors than a narrow beam. Finally, by comparing the results for $\delta = \lambda/40$ and $\delta = \lambda/80$ it is seen that the gain loss is approximately proportional to the distortion amplitude. This is in contrast to the focused reflector where the peak gain loss is proportional to the surface error squared.

4 Multibeam antenna with paraboloidal reflector

In the previous sections the shaping of the radiated beam was obtained by shaping the reflector surface. An alternative method to generate a shaped beam is to retain the paraboloidal reflector and to use a cluster of feed elements. The amplitude and phase excitations of each feed are then optimized in order to maximize the minimum gain within the coverage area. In the present context it is of special interest to investigate whether the multibeam antenna is more or less sensitive to surface errors than the surface shaped antenna. This comparison will be carried out in the following for the case of $\theta_b = 4^\circ$ and $\delta = \lambda/40$. 
The multibeam antenna geometry is \( f = D = 50\lambda \) and the feed array is designed by standard methods. It contains 81 circular elements 1.05\( \lambda \) in diameter and arranged in a hexagonal grid. The total feed cluster has a diameter of about 10\( \lambda \). After optimization of the feed excitations the minimum coverage gain becomes 27.29 dBi which is very similar to the 27.59 dBi obtained for the surface shaped antenna.

The paraboloidal surface of the multibeam antenna is now subject to the same 40 cases of surface distortions as were used for the surface shaped reflector and the coverage gain loss is compared in Figure 6. It is seen that the variation with the node spacing \( c \) is very similar for the two, but the maximum loss for the multibeam antenna is reduced to about two thirds compared to the surface shaped antenna. It is also significant that the rate of decrease for large \( c \) is much slower for the multibeam antenna.

![Figure 6](image)

**Figure 6** Coverage gain loss vs. node spacing \( c \), \( D/\lambda = 50 \), coverage: \( \theta_b = 4^\circ \), distortion amplitude: \( \delta = \lambda/40 \).

5 Conclusions

It has been shown that surface errors, which would give a peak gain loss of only 0.1 dB for a focusing antenna, may reduce the coverage gain for a shaped beam antenna by 2 dB or more. The gain loss is slightly smaller for a multibeam antenna than for a surface shaped antenna.

The results presented refer to a reflector diameter of \( D = 50\lambda \). It is, however, possible to scale these results in the following way: if \( D/A \) for an antenna with a coverage region \( \theta_b \) is increased by a factor \( F \) then the coverage gain loss will remain the same if the correlation distance is increased by \( F \) and the coverage region is reduced by \( F \).

For a realistic reflector it may be difficult to estimate the surface error correlation distance. Thermal distortions, creep and moisture absorption will normally give rise to slowly varying distortions. The manufacturing tolerances, by numerical milling for example, are normally very rapidly varying. For a panel reflector with adjustable panels the adjustment errors will have a correlation distance which is equal to the panel size.

The results presented have been obtained for antennas with rotational symmetry (before distortions). It is believed, however, that the results are equally valid for offset reflectors. The results can also be used, at least as a guidance, to non-circular shaped beams in which case the parameter \( \theta_b \) should be the radius of the circle on the far-field sphere which covers the same area as the actual shaped beam.

The present paper has been dealing with reflector surface errors and their impact on the far field for a shaped beam antenna. However, a very similar procedure can be used to assess the required tolerances for the reflectors of a compact range.