Quasi-optical beam waveguide analysis using frame based Gaussian beam expansion

Torsten Bondo[1] and Stig Busk Sørensen
Ticra, Laederstraede 34, DK-1201, Copenhagen

Abstract
A frame based Gaussian beam expansion method which can be used for analysis of quasi-optical beam waveguides is presented. The method is tested for the scattered field of an ellipsoid at 321 GHz and 3.21 THz.

Keywords: Quasi-optics, Beam waveguide, THz, Gaussian beams, PO, frames.

Introduction

For reflector antennas and mirrors with diameters less than 250 wavelengths Physical Optics (PO) is a fast and accurate calculation method for determining the scattered fields. In optics where the diameter of the mirrors are much larger than a thousand wavelengths ray tracing methods as Geometrical Optics (GO) are efficient ways of determining the scattered fields. However, at Quasi-optical frequencies the mirrors are typically in the order of a thousand wavelengths in diameter and in this range neither PO nor GO methods suffice. The computational time for PO increases with the frequency to the fourth power and GO methods become inaccurate or impractical. A fast analysis method useful for this wavelength range is the Gaussian beam analysis method.

Imbriale et. al. [1] expand the field in a beam waveguide in Gauss-Laguerre beams, which are used to propagate the field from a reflector A to a reflector B. On the reflector B the currents are calculated by the PO approximation. The scattered field of the reflector B is found by making a new expansion in Gauss-Laguerre modes on an output plane in front of the reflector B. This involves the calculation of orthogonality integrals with respect to the reflected field which is unknown at this stage. By use of the reciprocity theorem the orthogonality integrals can be transformed to surface integrals on the reflector B that only involve the known surface currents. The method is fast, but it has the major disadvantage that the Gauss-Laguerre expansion is only accurate in the paraxial region so that diffractions from the reflector are not accurately described.

Parini et. al. [2] use a frame based expansion [4] consisting of fundamental Gaussian beam modes that are shifted and rotated in space. Reflector scattering is computed by GO and GTD using Gaussian beam diffraction techniques. This gives the scattered field on an output plane where a new frame expansion is made. In contrast to [1] the method is also able to compute non-paraxial fields since only the fundamental Gaussian beam modes are used in the expansion. The major drawback is the use of GO and GTD for computation of the scattered field which involves ray-tracing and may be inaccurate if the surface or rim has a complicated shape (e.g. a rectangular rim).

In this paper a method is presented which combines the features of [1] and [2] avoiding the major disadvantages of the two methods. The frame based expansion that consists of the fundamental Gaussian modes that are shifted and rotated in space is used to describe the field, and the reciprocity theorem is used instead of the GO+GTD analysis.

The method has been implemented in a computer program for analysis of a sequence of two 3D reflectors. If a sufficiently large number of expansion functions is included the accuracy of the new method is comparable

[1]Correspondence: email tb@ticra.com, web: www.ticra.com

to PO also outside the main beam but the computation time may then be longer than for PO. However, close to the main beam a good accuracy can be obtained with a small number of expansion functions and with a considerable saving of computation time in comparison to PO. Especially for computing the near-field at THz frequencies the method proves to be much faster than PO.

Frame based expansion

In [3] a uniform 2D aperture field is expanded in fundamental Gaussian beam modes that are shifted and rotated over the aperture plane. This method can be generalized to an arbitrary aperture distribution in 3D and will be used in the next section for an ellipsoidal reflector antenna. The mathematics of this method is described in [4] and involves the concept of windowed Fourier transforms and frame based expansion.

In 2D the expansion functions on the aperture plane are defined by:

$$g_{mn}(t) = \pi^{-1/4} e^{imp_0 t} e^{-\frac{1}{2}(t-nq_0)^2}. \tag{1}$$

This corresponds to Gaussian beams linearly shifted and rotated in space. n, m are integers that define the translation and the rotation of a gaussian beam, respectively and p_0, q_0 are characteristic constants that define the form of the function.

An arbitrary aperture field $f(t)$ can then be expanded in the series:

$$f(t) = \sum_{mn} c_{mn} g_{mn}(t) \tag{2}$$

where

$$c_{mn} =< \tilde{g}_{mn}, f >= \int_{-\infty}^{\infty} \tilde{g}^*_{mn}(t) f(t) dt. \tag{3}$$

The function $\tilde{g}_{mn}$ is the dual function to g_{mn} in this paper called the frame function. It can be computed from g_{mn} as described in [4].

In this way an arbitrary aperture field is expanded in Gaussian fundamental beams which can be propagated to the next reflector in the beam wave-guide.

Computation of the reflected field

A mathematical surface S is considered which provides an aperture plane and encloses the reflector currents $\bar{J}_1$, see Figure 1. $\bar{E}_1, \bar{H}_1$ are generated by the currents $\bar{J}_1$ and the fields $\bar{E}_2$ and $\bar{H}_2$ are Maxwellian fields propagating in a region with no sources. The fields $\bar{E}_2$ and $\bar{H}_2$ propagate in the opposite direction of $\bar{E}_1, \bar{H}_1$. Assuming that S is an infinite plane orthogonal to the z-axis at $z = 0$ the reciprocity theorem reduces to:

$$\oint_S (\bar{E}^t_1 \times \bar{H}^t_2 - \bar{E}^t_2 \times \bar{H}^t_1) \cdot d\bar{s} = \int_R (\bar{E}_2 \cdot \bar{J}_1) \cdot ds \tag{4}$$

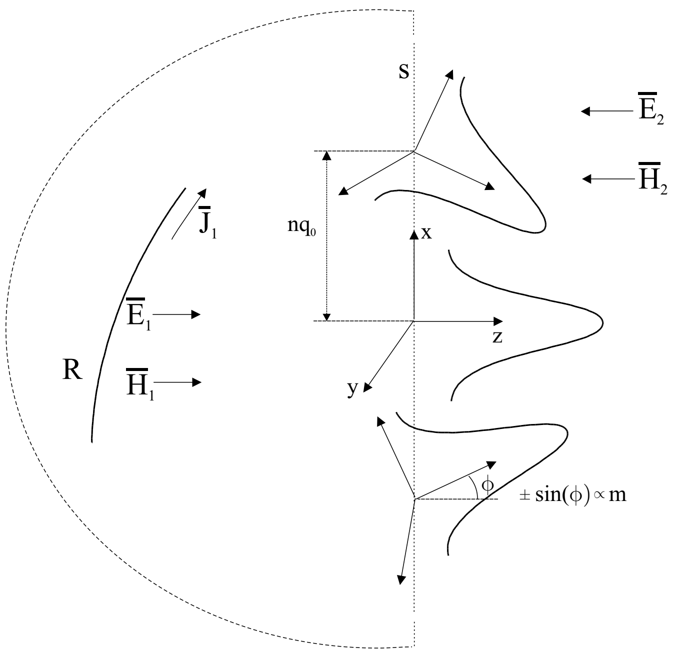

Figure 1: Illustration of the use of the reciprocity theorem and frame functions.

By a suitable choice of $\bar{E}_2$ and $\bar{H}_2$ the left hand side of (4) reduces to the integral in (3) such that the expansion coefficients c_{mn} can be computed from the right hand side of (4). The currents J_1 are the known PO currents on the reflector.

Hence, a method is constructed which uses the known PO currents on the reflector to compute a set of frame coefficients defined on an output plane by means of the reciprocity theorem. Once the coefficients are known the field value at any distance can be reconstructed as a sum of frame functions on the output plane weighted with the frame coefficients.

Results for a 3D ellipsoidal reflector

To evaluate the speed and accuracy of the frame based expansion method a comparison to PO at different frequencies and in different output planes is made.

A beam waveguide is considered which is operated at two frequencies 321 GHz and 3.21 THz. A coordinate system origin is placed at the reflection point of the ellipsoid where the centre ray combining the two focal points has a $90°$ reflection angle (see Fig. 2). In this coordinate system the two focal points are located in 63.5 mm along the negative x-direction and at 104.2 mm along the positive z-direction. The projection of the rim of the ellipsoid onto the plane orthogonal to the output direction along the z-axis, is a circle with diameter D=46.5 mm.

The product $p_0 * q_0$ must be $< 2\pi$ to ensure a stable expansion [4]. We have chosen $p_0 * q_0 = \pi/2$ with $q_0 = 1.5$ (see (1)) because it gives a simple frame function similar to the Gaussian fundamental function, but other values of p_0 and q_0 can also be used (and may prove to be more efficient - this will not be discussed in this paper).

The equations (1), (2), (3) must take into account the actual geometry of the system. Therefore, a scaling variable $x = Lt$ is introduced, where $L = w_0/\sqrt{2}$ and w_0 is the waist radius of the output beam. By this choice of scaling the width of the expansion frame functions matches the width of the output Gaussian beam in the waist. At the waist plane for 321 GHz, $L = 2.32$ mm and for 3.21 THz, L =0.245 mm. The input Gaussian

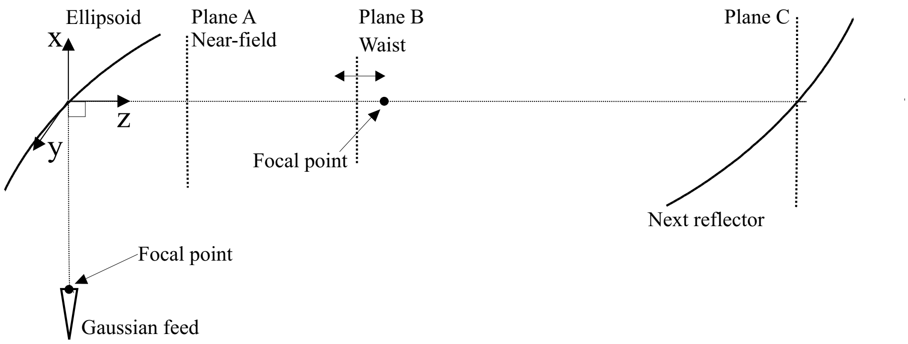

Figure 2: Geometry of the beam waveguide system.

beam has $w_0 = 2.12$ mm at 321 GHz and $w_0 = 0.212$ mm at 3.21 THz.

The Gaussian feed is radiating from 60 mm and 63.5 mm along the negative x-axis at 321 GHz and at 3.21 THz, respectively [1]. The beam is then scattered by the ellipsoid and the field is evaluated in three planes A, B and C (see Figure 2) :

A. The near-field plane of the reflector in a distance of 40 mm along the z-axis. The field is evaluated from $x = -25$ mm to $x = 25$ mm which is approx. the size of the projected diameter ($D = 46.5$ mm).

B. The plane located in the waist of the reflector which is placed at a distance of 88.7 mm for 321 GHz and in a distance of 103.9 mm for 3.21 THz along the z-axis. This is also the plane used to calculate the frame coefficients. The main lobe and approx. 4-5 side lobes are plotted for both frequencies along the x-axis.

C. The plane orthogonal to the ray along the z-axis at the reflection point of the next reflector in the beam waveguide at a distance of 240 mm along the z-axis. The field is evaluated from $x = -40$ mm to $x = 40$ mm which is the size of the projected diameter of the next reflector ($D = 80$ mm).

The speed of the frame method can be compared to the speed of the PO method by comparing the number of PO current elements to the number of frame coefficients.

A total of 3969 frame coefficients is used in the frame based expansion. In the expansion the n-number of the frame coefficients denotes the translation of the Gaussian beam and the m-number the rotation of the Gaussian beam (see (1) and Figure 1).

Therefore, the n-number must be chosen such that the Gaussian beams cover the field to be expanded. Furthermore, the m-number determines the degree of rotation of the Gaussian beams. By increasing or decreasing this number the area in which the radiated field is converged increases or diminishes.

[1] At 3.21 THz the Gaussian beam is radiating from the focal point

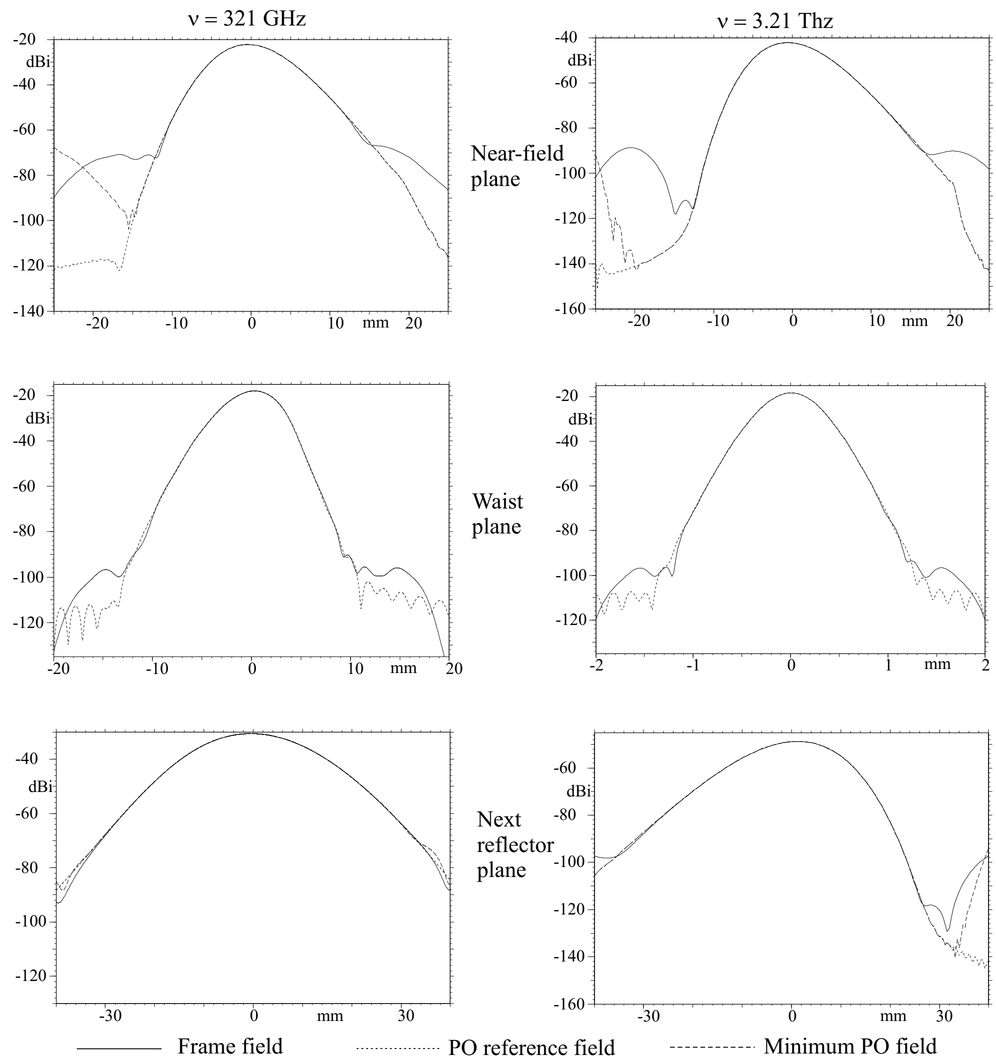

Figure 3: Near field at Plane A, waist field at plane B and field at the next reflector at plane C for 321 GHz and 3.21 THz (see Fig. 2 for set-up). Three types of fields are plotted: the frame field, a converged PO reference field and a PO field where the minimum number of PO points that ensures field convergence 45 dB below peak is used. The normalization of the curves is such that power flux in dB per unit area can be calculated by adding $20\log_{10} k$. Only one PO curve is shown in Plane B, as the minimum PO solution and the reference PO curve overlap in this field region.

	Near field plane	Waist plane	Next reflector plane
PO 321 GHz	5186	961	315
PO 3.21 THz	507348	687	21377
Frame 321 GHz	3969	3969	3969
Frame 3.21 THz	3969	3969	3969

Figure 4: Table of the number of PO current elements for the minimum PO solution and the number of frame functions at 321 GHz and 3.21 THz that is used to obtain the results presented in Figure 3.

In Figure 3 the results of the tests are shown and table 4 contains the number of PO current elements and the number of frame functions at 321 GHz and 3.21 THz that are used to obtain the results presented in Figure 3. Two types of PO solutions are shown: The reference PO solution where a sufficiently large number of PO points is used to obtain field convergence over the whole output plane area and the minimum PO solution where the number of points are just sufficient for convergence down to -45 dB below the peak value.

The results show that a constant number of frame coefficient can be used for both frequencies on all output planes. In all cases the frame results are converged down to approx. -45dB below peak value. This stable results should be compared to PO where the necessary number of current elements is both frequency and position dependent. Especially in the near field at the high frequency a very high number of PO points must be used to make the field converge down to -45 dB. PO is, however, a fast method for computing the field in the waist of the beam waveguide, where the field is in phase. For the plane of the next reflector PO is very fast at low frequencies but becomes slow at high frequencies.

Conclusion

As a beam waveguide analysis tool at THz frequencies a frame based Gaussian beam method using a combination of two different techniques [1, 2] has been presented.

The method is frequency independent and only the number of the Gaussian beam functions determines the computational time. The accuracy of the method increases with m and n and can reach PO accuracy. However, for high m and n numbers the method can no longer compete with PO in speed. Also for simple field calculations where the field is in phase on the output plane, PO is a faster method.

For computing the near field the method proves to be very successful at THz frequencies, where PO is very time consuming when the near-field plane is close to the scattering object.

The method also has the advantage, that the fundamental Gaussian beam modes used in the expansion method are not limited to the paraxial region and a general reflector shape and rim can be handled in a beam waveguide system.

We believe that the frame based Gaussian beam method with the right tuning (eg. choice of the grid parameters p_0, q_0) can work as an intermediate accurate and stable alternative to the accurate, but in some cases slow method of PO and the fast and more unstable method of GO+GTD.

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