On the use of FFT in surface shaping of contoured beam antennas

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Abstract: A comparison of the Fast Fourier Transform (FFT) and direct numerical integration to solve the PO integral is presented, with special emphasis on the achieved accuracy and the CPU time consumption. The concept of second order FFT is introduced. It is shown that the accuracy of FFT is not compatible with the accuracy requirements expected by analysis codes for satellite communication antennas.

Introduction

The FFT method has often been proposed to solve the PO integral in a fast and accurate way, as an alternative to the straightforward numerical integration of the induced currents on a reflector surface. The method has received particular attention in the design of shaped reflectors for contoured beams, since in this case the radiation from the antenna must be calculated repeatedly at many far-field points as part of an iterative optimization scheme [1]. Because the PO integral cannot be formulated directly as a Fourier integral it is necessary to introduce approximations which affect the accuracy of the calculated field. The impact of these approximations has been studied, and is viewed in relation to an accuracy requirement of ±1 dB at a level 40 dB below the antenna peak gain in the following.

FFT approximation of the PO integral

If we suppress the factor $e^{-jkr}(kr)^{-1}$, the PO far field can be written as

$$\overline{E}_{\mathrm{far}}(u,v) = -\frac{j\zeta}{4\pi}\Big[\iint \overline{J} e^{jk(xu+yv+zw)} k^2 ds\Big]_{\mathrm{tan}}, \qquad (1)$$

where $w^2 = 1-u^2-v^2$ and the subscript 'tan' denotes the tangential component. The integral contains the factor e^{jkzw} which is incompatible with the FFT, where the integrand must be of the form $A(x,y)e^{jk(xu+yv)}$. To bring the integral into the required form the exponent is rearranged as

$$e^{jkzw} = e^{jkz}e^{-jkz(1-w)} = e^{jkz}\Big(1 - jkz(1-w) - \tfrac{1}{2}\big(kz(1-w)\big)^2 + \ldots\Big). \qquad (2)$$

The Taylor series is valid for a narrow beam where $w = \cos\theta \cong 1$. The far field can thus be calculated as a series of FFT's:

$$\overline{E}_{\mathrm{far}} = \overline{E}_{\mathrm{far},1} + \overline{E}_{\mathrm{far},2} + \ldots \quad , \qquad (3)$$

where

$$\overline{E}_{\mathrm{far},1}(u,v) = -\frac{j\zeta}{4\pi}\Big[\iint \Big(\overline{J}e^{jkz}\Big) e^{jk(xu+yv)} k^2 ds\Big]_{\mathrm{tan}}, \text{ and} \qquad (4)$$

$$\overline{E}_{\mathrm{far},2}(u,v) = -\frac{j\zeta}{4\pi}\big(-jk(1-w)\big)\Big[\iint \Big(\overline{J}ze^{jkz}\Big) e^{jk(xu+yv)} k^2 ds\Big]_{\mathrm{tan}}. \qquad (5)$$

The first integral is what is normally included in the FFT evaluation of the PO integral whereas the second term, the inclusion of which we will refer to as second order FFT, has been described in the literature as part of the Jacobi-Bessel approach [2].

It can be shown that the FFT approximation can be further improved by using a tilted aperture plane (which contains the edge of the reflector) as support for the FFT rather than the projected aperture plane orthogonal to the boresight direction [3]. This will further be addressed in the test example.

FFT input and output points

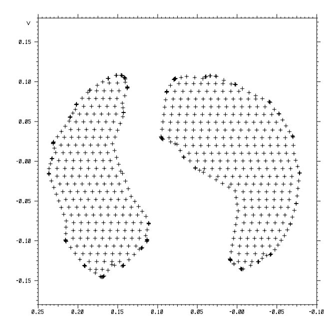
In FFT the input and output points must be arranged in regular rectangular grids. A rectangular grid of integration points on the reflector aperture does not fit very well to a circular aperture which means that the density of points must be higher than for standard Gaussian integration in a polar grid. For reflector shaping the desired output points are typically a number of irregularly distributed points so that the far field in these points must be found by interpolation in the regular FFT output grid in the uv-plane. We will consider two types of interpolation.

The first type is cubic interpolation. For cubic interpolation the necessary spacing of the output points is $\lambda/(4D)$ which means that the side length of the integration rectangle should be increased to 4D with zeroes assigned to the integration points outside the reflector. In order not to decrease the accuracy, the number of integration points along each side must be increased from N to 4N. This, of course, leads to a very significant increase of the computation time and memory requirements.

The second type uses Whittaker interpolation. This interpolation technique requires only a slight over-sampling by increasing the side length of the integration rectangle and the number of points by a factor of 1.2. However, the interpolation involves the computation of a number of trigonometric functions which are more time consuming to calculate than rational functions. It has been found advantageous first to use Whittaker interpolation to compute the field in a grid 4 times denser than the original FFT output grid and then compute the field at the irregular far-field points by cubic interpolation. Hence the second type is referred to as Whittaker-cubic interpolation.

Test example

As a representative test example a typical Intelsat antenna has been chosen. The antenna is a shaped single reflector antenna with circular aperture. The antenna has a focal length of 3.3 m, a diameter of 3.3 m, an offset distance of 1.85 m and is operated at 4.0 GHz. The hemi-beam coverage areas are shown in the figure to the right where the antenna radiation is maximised in the eastern hemi beam while the side-lobes are suppressed in the western hemi beam.



A reference result has been computed for the exact integral (1) with a fine Gaussian integration grid such that the absolute accuracy in each of the output stations is better than 10^{-4}.

In order to judge the accuracy of the FFT field calculation we will consider the accuracy required by Intelsat which is a maximum error of 1 dB at a field level 40 dB below the peak level. This is translated into a difference between the amplitude of the computed value and the amplitude of the reference solution of approximately -58.8 dB relative to the peak level of the reference solution.

Accuracy :

The impact of the approximations made to bring the PO integral into the FFT form on the achieved accuracy is assessed below. The 1.–order (4) and the 2.–order (4)+(5) approximations of the PO integral (1) are calculated and compared to the reference solution. Results are obtained with and without the use of the tilted aperture approach [3].

	Without tilted aperture		Tilted aperture	
	1.–order	2.–order	1.–order	2.–order
Eastern cov. alone	-29.32 dB	-39.99 dB	-39.61 dB	-54.93 dB
Both coverages	-29.32 dB	-36.90 dB	-39.61 dB	-53.23 dB

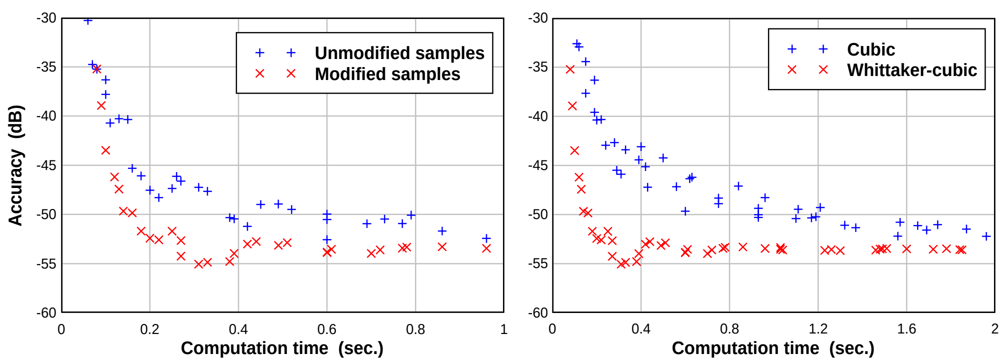
We conclude that the accuracy of FFT will be rather poor unless the 2.–order term is included and the aperture plane is tilted. The accuracy will then be sufficient for many applications although the Intelsat criterion is not met.

Computation time for FFT:

Next, the computation time is considered. For the FFT results, the 2.–order approximation and the tilted aperture approach are used (the tilted aperture plane approach gives no penalty with respect to computation time whereas the 2.–order FFT will double the computation time compared to the 1.–order FFT). Calculations have been performed for an increasing number of integration points, and for each calculation we plot the accuracy obtained as a function of the computation time.

As shown in Figure 1(a), the computation time to reach a specific accuracy of the FFT can be significantly reduced by modifying the samples of the surface currents close to the reflector rim. The FFT method implicitly weights all samples inside the rim by the area $\Delta x \Delta y$ of a 'unit cell' of the FFT grid and samples outside the rim by zero. Instead, for samples close to the reflector rim we define the weight factor as the actual area of the unit cell inside the rim.

Also it has been found that a given accuracy level is reached faster using the Whittaker-cubic interpolation than using standard cubic interpolation, c.f. Figure 1(b).



(a) (b)

Figure 1: Influence on computation time of the modified samples of the surface currents and of the interpolation.

Gaussian integration versus FFT

Finally, in Figure 2, the Gaussian integration and the FFT method are compared. In the FFT computations, the tilted aperture, the modified samples and the Whittaker-cubic interpolation are used.

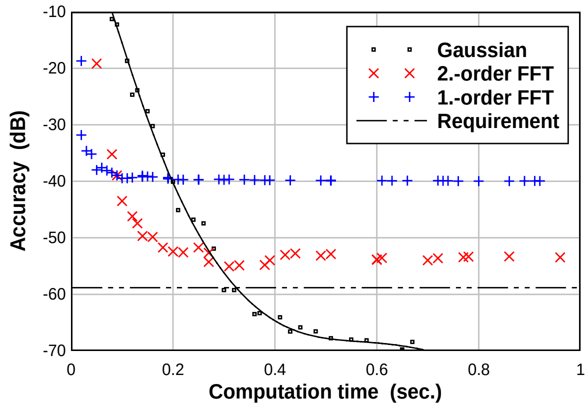


Figure 2: Accuracy vs. computation time for the Gaussian and FFT methods.

Conclusions

The study has shown that in order to meet standard accuracy requirements for spacecraft antenna analysis it is necessary to use at least 2.–order FFT as alternative to Gaussian integration. This in turn means that the time-saving by using FFT becomes smaller than usually obtained.

It is emphasised that the computation times shown in the paper are only for the calculation of the PO integral and do not involve the time for the calculation of the PO currents. If the incident field calculation is time-consuming, e.g. the field from a subreflector, the Gaussian integration has an important advantage as it needs fewer integration points than FFT. For the example considered the Gaussian integration needs 4.5 times fewer integration points than the FFT on the reflector surface.

Also, in reflector shaping it is necessary to compute gradients of the far field with respect to the surface variables. By using a spline representation of the surface and Gaussian integration this can be done very conveniently by only repeating the integration over the local support of the corresponding B-spline function, leading to a significant time saving. With FFT this time saving is not possible.

Consequently, we conclude that the FFT does not have an advantage compared to the direct numerical integration for the present purpose.

The study also revealed that the computation time to reach a specific accuracy of the FFT can be significantly reduced by modifying the samples of the surface currents close to the rim. Furthermore, it was found that FFT combined with Whittaker-cubic interpolation is faster than FFT combined with the standard cubic interpolation.

For a larger antenna, measured in wavelengths, the conclusions reached above may be different. Here FFT will probably be even faster than Gaussian integration, but on the other hand the phase approximation used by FFT becomes more crude resulting in a less accurate far field.

References

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