Design Tool for High-Performance Rotationally Symmetric Reflector Antennas

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1. Introduction

Rotationally symmetric reflector systems are attractive candidates for realizing compact high-gain antennas with low manufacturing costs, low sidelobes, and low cross polarization. The typical application areas are point-to-point communication links, satellite terminals, and radar systems. These compact systems often employ two reflectors in a classical axially displaced reflector configuration [1], or alternatively, a single reflector with a backward radiating hat feed [2] or splash-plate feed [3]. A common feature of these compact systems is that they include a very tight integration of feed, subreflector, dielectric support structure, and main reflector, which leads to a resonant structure that cannot be analyzed and designed using ray-optical methods. Instead, a full-wave model is needed to analyze the performance of the system with sufficient accuracy and consequently, the full-wave model is also needed when numerically optimizing the antenna performance. Commercially available software for general 3D problems lead to prohibitively long run-times that do not allow numerical optimization. Instead, the rotational symmetry of the structure has been used to formulate the Body-of-Revolution Method of Moments (BoR-MoM) [4], including various formulations for composite metallic/dielectric structures [5]. The MoM problem in [4] was discretized using triangular basis functions and this formulation has been inherited by most later works, including [5]. This low-order discretization typically requires 15 unknowns per wavelength to achieve accurate results, leading to a typical analysis time of 30-60 seconds per frequency for a compact hat-feed antenna. This analysis speed is sufficient for many purposes, but not for full optimization of the combined system including the surface shape of the main reflector, the features of the feeding waveguide, the shape of the dielectric support, and the fine details on the feed hat or the subreflector.

In this paper, we present an efficient higher-order BoR-MoM in which the unknown surface current is discretized using hierarchical Legendre basis functions [6], that are adapted to the circular symmetric geometry. Furthermore, the shape of the generatrix is represented by up to 4th order curvilinear line segments that approximates curved surfaces with high accuracy. The combination of a higher-order polynomial current approximation and the smooth geometry implies that the number of unknowns is reduced to approximately 4 per wavelength, i.e., approximately a four-fold reduction compared to the low-order case. This reduction leads to a dramatic speedup such that a full-wave analysis of typical rotationally symmetric antennas can be accomplished in 1-2 seconds on a laptop.

The new higher-order BoR-MoM has been rigorously combined with a mode-matching tool for circular horns which implies that complex feeds, such as corrugated feeders, can also be analyzed in a couple of seconds. Furthermore, this hybrid mode-matching/higher-order MoM analysis engine is integrated in an optimization framework for rotationally symmetric reflectors and feeders. This design tool integrates all the required functionality in a single tool which is demonstrated by a design example of a compact antenna with a very small aperture.

2. Higher-Order BoR-MoM for Fast Full-Wave Analysis

The BoR-MoM has been formulated previously in several works, e.g., for conducting objects [4], for dielectric objects [7], and for composite metallic/dielectric objects [5]. All these works have employed triangular basis functions on flat curve segments which require a relatively large number of unknowns to achieve convergence. In this work, we employ the same continuos integral equation as in previous works, e.g., that of [5], but the equation is discretized with higher-order basis functions and curved segments of up to 4th order. The basis functions applied here are those of [6] which have been adapted to the present case with rotational symmetry. The electric and magnetic surface currents on each curve segment are expanded as

$$\mathbf{X} = \sum_{m=0}^{M^\phi} \sum_{n=0}^{N^t} a_{mn}^{t,e} \mathbf{B}_{mn}^{t,e} + a_{mn}^{t,o} \mathbf{B}_{mn}^{t,o} + \sum_{m=0}^{M^\phi} \sum_{n=0}^{N^t-1} a_{mn}^{\phi,e} \mathbf{B}_{mn}^{\phi,e} + a_{mn}^{\phi,o} \mathbf{B}_{mn}^{\phi,o}, \mathbf{X} = \mathbf{J}, \mathbf{M} \tag{1}$$

where $a_{mn}^{t,e}$, $a_{mn}^{t,o}$, $a_{mn}^{\phi,e}$, and $a_{mn}^{\phi,o}$ are unknown coefficients, N^t is the polynomial expansion order along the generatrix, M^ϕ is the azimuthal mode index, and $\mathbf{B}_{mn}^{t,e}$, $\mathbf{B}_{mn}^{t,o}$, $\mathbf{B}_{mn}^{\phi,e}$, and $\mathbf{B}_{mn}^{\phi,o}$ are $t-$ and ϕ-directed vector basis function defined as

$$\mathbf{B}_{mn}^{t,\binom{e}{o}}(t,\phi) = \frac{\mathbf{a}_t}{\mathcal{J}_s(t,\phi)} \widetilde{P}_n(t) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}, \quad \mathbf{B}_{mn}^{\phi,\binom{e}{o}}(t,\phi) = \frac{\mathbf{a}_\phi}{\mathcal{J}_s(t,\phi)} P_n(t) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}, \tag{2}$$

where $\mathbf{a}_t = \partial \mathbf{r}/\partial t$, $\mathbf{a}_\phi = \partial \mathbf{r}/\partial \phi$, and $\mathcal{J}_s(t,\phi) = |\mathbf{a}_t \times \mathbf{a}_\phi|$. In Eq. (2), the polynomials $P_n(t)$ along the direction transverse to the current flow are chosen to be Legendre polynomials due to their nice orthogonality properties. In the direction along the current flow, Legendre polynomials are not appropriate since they would not allow the normal current continuity to be enforced. Instead, the modified Legendre polynomials

$$\widetilde{P}_n(t) = \begin{cases} 1 - t, & n = 0 \\ 1 + t, & n = 1 \\ P_n(t) - P_{n-2}(t), & n \geqq 2 \end{cases} \tag{3}$$

are used. The polynomials in (3) are zero at $t = \pm 1$ for $n > 1$ which implies that the high-order terms do not contribute to the current continuity. The two lowest order polynomials $1 \pm t$ are each contributing to the continuity at one of the edges at $t = \pm 1$. These functions can be matched with the corresponding functions on the neighboring curve segments, or alternatively, the functions with either $n = 0$ or $n = 1$ can be left out at segments with external nodes to force the current to zero. The modal expansion order, M^ϕ, should be adjusted to the specific problem and for a problem excited by the fundamental TE_{11} mode it is sufficient to include $m = 1$. When the basis functions are used, the typical number of unknowns required for convergence drops to about 4 per wavelength, implying that the matrix equation is easily stored and inverted, even for very large structures such as reflectors with a diameter of several hundred wavelengths. The bottleneck for smaller problems is usually the matrix fill time which, however, can be dramatically reduced by using analytical techniques [8].

3. Design Tool Based on a Hybrid Mode-Matching/MoM Solution

The higher-order MoM presented above is ideal for open problems with radiation in free space. However, the feed horn is often a complex waveguiding device, e.g., a corrugated horn, where a circular mode-matching algorithm is the fastest option [9]. For this reason, the scattering parameters of the exterior part, i.e. the reflectors, the horn exterior, the support structures, etc, are extracted using higher-order BoR-MoM and a waveguide port model similar to that of [10], but adapted to the present case of higher-order basis functions, rotational symmetry, dielectric material, and an unbounded radiation problem. The full scattering matrix of the antenna is then obtained by cascading the individual matrices of the horn and the exterior part. This hybrid Mode-Matching/Higher-Order BoR-MoM analysis has been integrated in a flexible design software that combines all the required capabilities in a single tool. The geometry can be defined in several ways with increasing level of details, ranging from a high-level model with few parameters to a fine-grained model with a large number of optimization variables. Reflector surfaces and horn profiles are conveniently defined as splines with user-specified and optimizable control points. The tool includes multiple optimization algorithms as well as several built-in optimization goals, e.g., return loss, directivity, side-lobe level, cross-polar radiation, co- and cross-polar pattern templates. In addition, the tool includes design wizards for the classical axially displaced reflector systems [1] which may serve as the starting point of a thorough numerical optimization.

4. Example Design of a Compact Antenna with Low Side-Lobes

The design tool is now tested with a practical antenna synthesis problem that has been published previously [11]. The antenna is a very small hat-feed reflector antenna with a main reflector diameter of 24 inches at 5.8 GHz, corresponding to 11.8λ. The requirements for this antenna are listed in Table 1. The feeding waveguide needs to be above cut-off and the small main reflector and the low sidelobe requirement limits the allowable size of the feed hat. These constraints make it a very challenging task to obtain a reasonable return loss and the solution of [11] therefore employs a small dielectrically filled waveguide and the solid dielectric also fills the space between the feed and the feed hat. With this configuration the pattern requirements are met but the return loss is 12 dB [11] when the main reflector is included.

The new design tool discussed in Section 3 has been used to design an alternative antenna of the same size but without dielectric filling in the waveguide. Instead, the feed hat is supported by a dielectric cone with a relative permittivity of 2.5. The feed hat is designed with a stepped profile and the main reflector is highly shaped to force the sidelobes down. The antenna was optimized using a total of 32 optimization variables and the geometry of the optimized design and the radiation pattern are shown in Figure 1. Table 2 lists the performance parameters of the designed example as well as the original design proposed in [11]. The example design presented here differs significantly from that of [11] by providing a higher directivity, lower sidelobes, a much higher return loss, and lower cross-polar radiation, while avoiding the need for a dielectrically filled waveguide.

Table 1: Design Requirements for a compact reflector system, from [11]

Frequency Range	5.725 GHz to 5.875 GHz
Aperture diameter	24 inches (0.61 m $\approx 11.8\lambda$)
Maximum VSWR	1.5:1 (return loss 14 dB)
Polarization	V or H
Antenna gain	27.5 dBi
Sidelobe level relative to peak	-20 dB
Maximum cross-polarization level relative to co-polar peak	-25 dB

Table 2: Performance parameters of compact reflector systems

	Design from [11]	Example from this paper
Directivity	28.4 dBi	29.4 dBi
Return loss	> 12 dB	>20 dB
Sidelobe level relative to peak	-25.9 dB	-26.0 dB
Maximum cross-polarization level relative to co-polar peak	-20.5 dB	-28.0 dB

5. Summary

This paper has presented a new higher-order BoR-MoM that provides at least an order of magnitude speedup compared to the standard formulation using triangular basis functions. The new algorithm allows a very fast full-wave analysis of rotationally symmetric reflector systems, including feeders, subreflectors, dielectric support structures, or radomes. The typical run-time is reduced to 1-2 seconds per frequency on a standard laptop. The new algorithm has been integrated in a dedicated design software, including mode-matching on circular horns, spline-shaping of reflectors, several high-level models and wizards, as well as powerful optimization routines. The capabilities of the tool were illustrated with a practical antenna design problem, in which a compact 11.8λ antenna was designed to meet the requirements in a previously published work [11]. The design presented here meets all the requirements and provides higher directivity, lower cross-polar radiation, higher return loss, as well as lower sidelobes compared to the design presented in [11]. These significant improvements can be attributed to the fast analysis algorithm, that allows the optimization procedure to seek the solution in a very large parameter space.

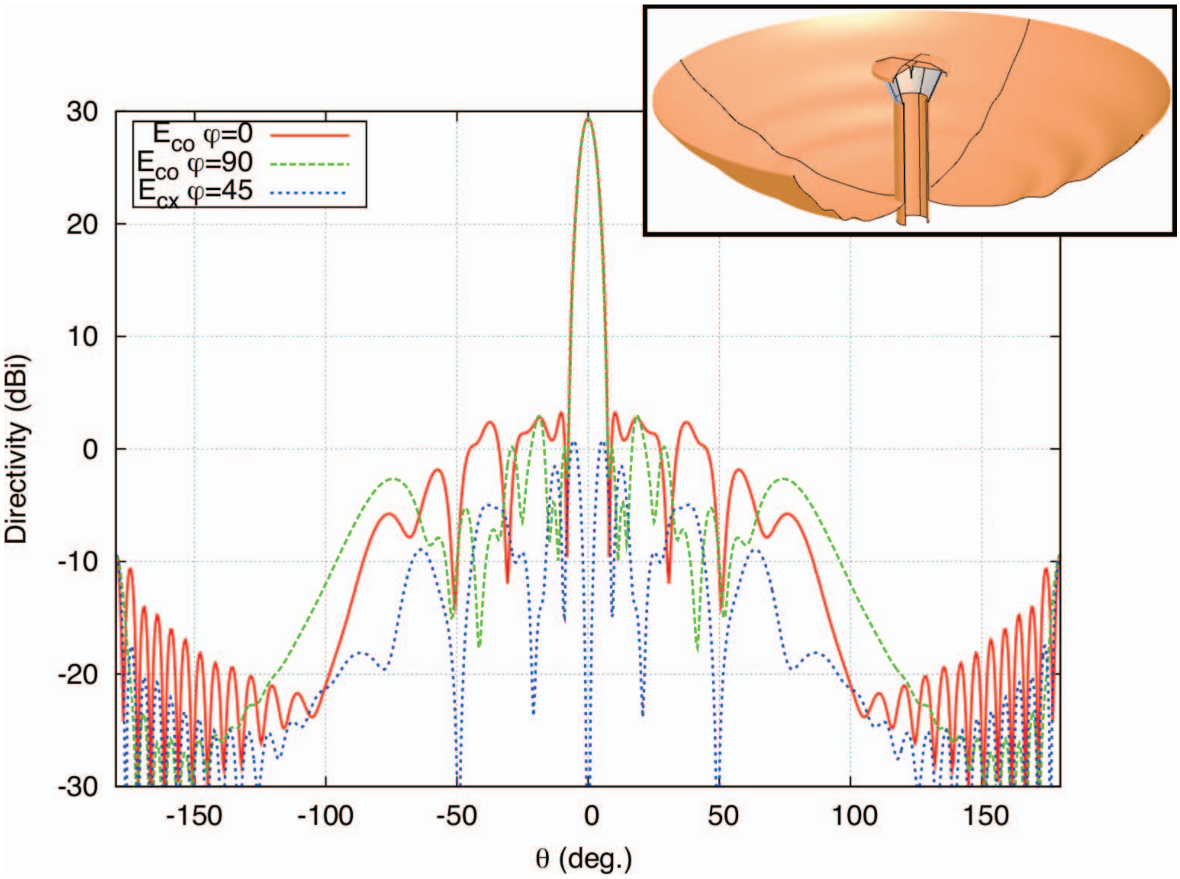

Figure 1: Radiation pattern of the 11.8λ compact rotationally symmetric reflector antenna. Insert: The optimized geometry with a 45 degree cut-out and the dielectric support structure is shown in blue.

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