

New Fast and Robust Modelling Algorithms for Electrically Large Antennas and Platforms

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Abstract—This paper presents two new modelling algorithms that was recently added to the commercially available GRASP software package for electrically large antenna and scattering problems. In particular, a new higher-order multilevel fast multipole solver (HO-MLFMM) provides very high simulation accuracy while requiring significantly less memory and CPU time than the commonly available low-order MLFMM. At the same time, the solver incorporates a generalised form of the underlying surface integral equations, which allows the use of non-connected meshes and enhances the robustness in real-life applications. As a further addition to GRASP, we outline a new Fast-PO algorithm which provide accelerated integration of Physical Optics currents as well as surface currents in general. The algorithm eliminates some of the limitations of previously published methods, e.g., by allowing observation points located well within the near-field region of the antenna without resorting to a much slower direct surface integration. The capabilities of the new algorithms are illustrated by examples involving a large reflector antenna ground station and a detailed model of a communication satellite.

Index Terms—method of moments, higher-order basis function, multi-level fast multipole method, fast physical optics, reflector antennas, platform interactions

I. INTRODUCTION

Solution of scattering and radiation problems continues to be a challenging area that may require significant computer hardware resources. Several software packages are available and offer general-purpose solvers based on full-wave or asymptotic solutions to Maxwell's equations. One such software package is GRASP which is commonly used for analysis of reflector antenna problems and other electrically large problems, e.g., scattering by satellite platforms. This paper describes two fast and accurate modelling algorithms that have recently been added to GRASP.

The most successful method for full-wave solution of electrically large antenna problems is the Multilevel Fast Multipole Method (MLFMM) [1] which is available in the majority of commercial software packages for antenna design and analysis. This widely available algorithm solves Maxwell's equations in the frequency domain and is typically based on a surface integral equation in which the currents are discretised using RWG basis functions on triangular domains. Several researchers have proposed higher-order (HO) basis functions, e.g. [2], that allow the number of unknowns to be reduced by a factor of 5 while maintaining the same accuracy as obtained using RWGs. However, higher-order basis functions result in poor performance in standard MLFMM implementations, see e.g. [3], and basis functions higher than 2nd order are generally

not available in commercial MLFMM solvers. In order to solve this problem, a revised MLFMM scheme, the so-called HO-MLFMM, was recently presented in [4]. This algorithm allows basis functions of very high orders to be used effectively in MLFMM. This approach has been implemented in a new HO-MLFMM solver that allows significant memory and CPU savings when compared to the widely available RWG-based MLFMM. This new solver is described in Section II below.

Integration of surface currents over electrically large surfaces is required in several applications, e.g., after solution of an MLFMM problem or when Physical Optics (PO) is used to compute surface currents on very large scatterers. In such cases, both the number of field output points and the number of current samples grow with the square of the frequency, therefor leading to $O(f^4)$ frequency scaling. A standard integration procedure becomes extremely time-consuming and a fast algorithm is in high demand. Several such algorithms have been published in the past, e.g., [5], but their performance is limited when observation points are located very close to the current samples. In addition, the existing algorithms have not demonstrated the accuracy needed for industrial applications. A new algorithm that solves these problems have just been introduced [6] in GRASP and the performance of this algorithm is demonstrated in Section III below.

II. EFFICIENT HO-MLFMM SOLVER

Higher-order MoM is very efficient and typically reduces the number of unknowns by a factor of 5 when compared to the commonly applied low-order MoM. Despite this reduction in unknowns, the memory and CPU requirements of MoM grow rapidly with frequency and an efficient MLFMM is needed. Multiple research groups have attempted to make MLFMM work together with higher-order MoM, but the algorithms presented so far did not work well with expansion orders higher than two. Recently, we presented a HO-MLFMM that enables the use of very high expansion orders and provides great savings in comparison to the widely available low-order MLFMM. This HO-MLFMM has been implemented in a new solver available in GRASP and the performance of this solver is studied in Section II-A below. The solver is based on a generalized set of surface integral equations which results in a very robust solver that works with defective and non-connected meshes, as illustrated in Section II-B. Finally, the HO-MLFMM solver is applied to a practical antenna problem in Section II-C.

A. Memory and CPU Performance of HO-MLFMM solver

The new HO-MLFMM solver works with both h - and p -refinement [2], implying that higher accuracy can be obtained by reducing the patch size h or by increasing the polynomial order p . Typically, a high polynomial order and large patches are applied in smooth regions of the scatterer whereas small patches and a low polynomial order are used to model small geometrical features. When MLFMM is applied with different expansion orders, there is no direct link between the number of basis functions and the solution error. Therefore, it is more illustrative to study the required resources versus the solution error since this shows how one can minimise the error for a given set of resources. In order to study the solution error we compare with an exact solution by computing the scattering from a sphere with a diameter of 50λ . A fixed polynomial order between 1 and 5 is applied on meshes with different patch densities and the memory, CPU time, and solution error are recorded for each run. The plot in Figure 1 shows the results for the memory performance for both a standard MLFMM [1] and the HO-MLFMM [4]. The following observations can be made:

- The standard MLFMM scheme (red curves) does not work well with higher-order expansion functions because the memory requirement grows when the order is increased. The lowest memory for a given accuracy is obtained by applying 2nd order basis functions ($p = 2$). This observation is in line with [3].
- When the standard MLFMM scheme is used, the memory requirement grows rapidly if high accuracy is desired.
- The HO-MLFMM algorithm provides great memory savings, even for first-order basis functions.
- With the HO-MLFMM approach (blue curves), the memory curve is almost flat and it is very cheap in terms of memory to ask for high accuracy.
- With HO-MLFMM (blue curves), all expansion orders higher than one results in roughly the same memory requirement to reach a desired error level.

The last item above may suggest that there is no significant benefit from using high expansion orders. However, this is not at all the case, which is apparent when observing the CPU time requirements for each matrix vector product reported in Figure 2. The following observations can be made:

- There is a slight CPU time penalty by using the new HO-MLFMM scheme relative to the traditional MLFMM scheme. However, the small CPU time penalty enables the large memory savings reported in Figure 1.
- There is a direct relation between the CPU time requirement and the expansion order. For a given RMS error, the fastest solution is always obtained by choosing the highest possible expansion order.
- The CPU time curve is almost flat for the highest expansion order. This implies that the time required for performing the matrix-vector product is virtually unchanged when a higher accuracy is desired.

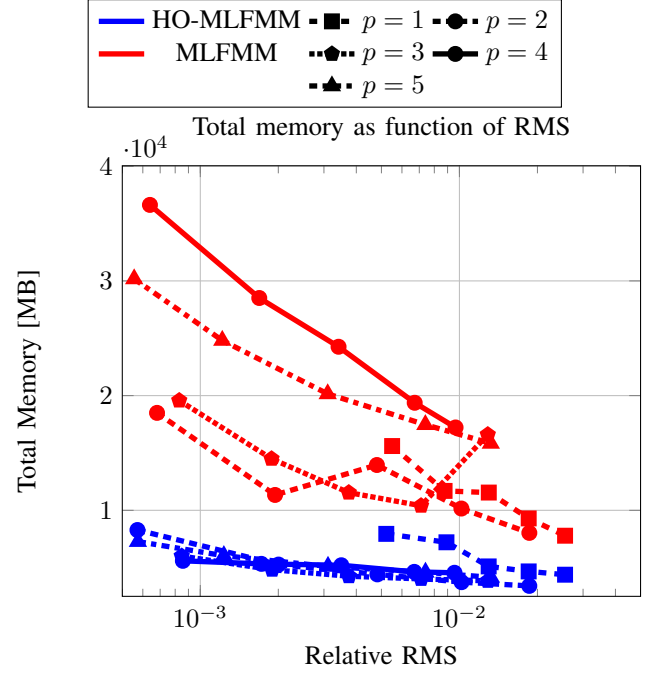


Fig. 1. The total memory for varying RMS error and polynomial order p , using CFIE, with the standard MLFMM algorithm and the modified HO-MLFMM algorithm.

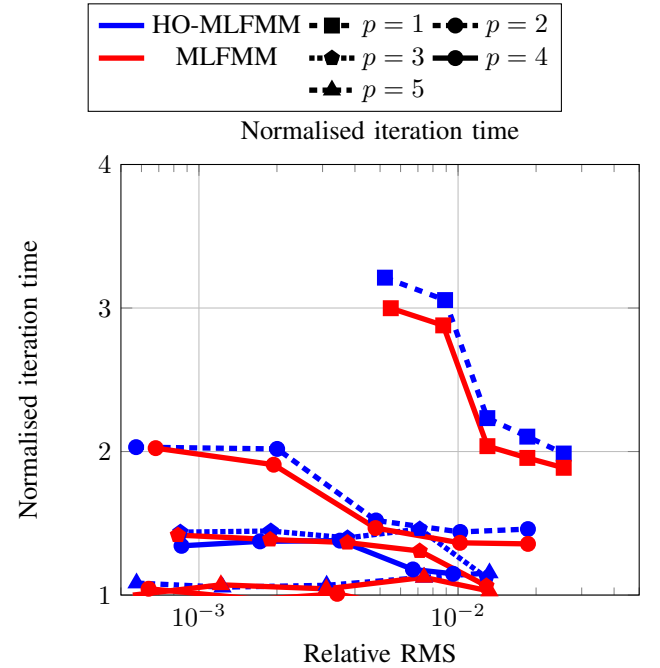


Fig. 2. Time per matrix-vector product normalised to the time for the fastest run. Each curve corresponds to a fixed polynomial order ($p=1, 2, 3, 4, 5$) and results are shown for the standard algorithm (red) and the modified algorithm (blue).

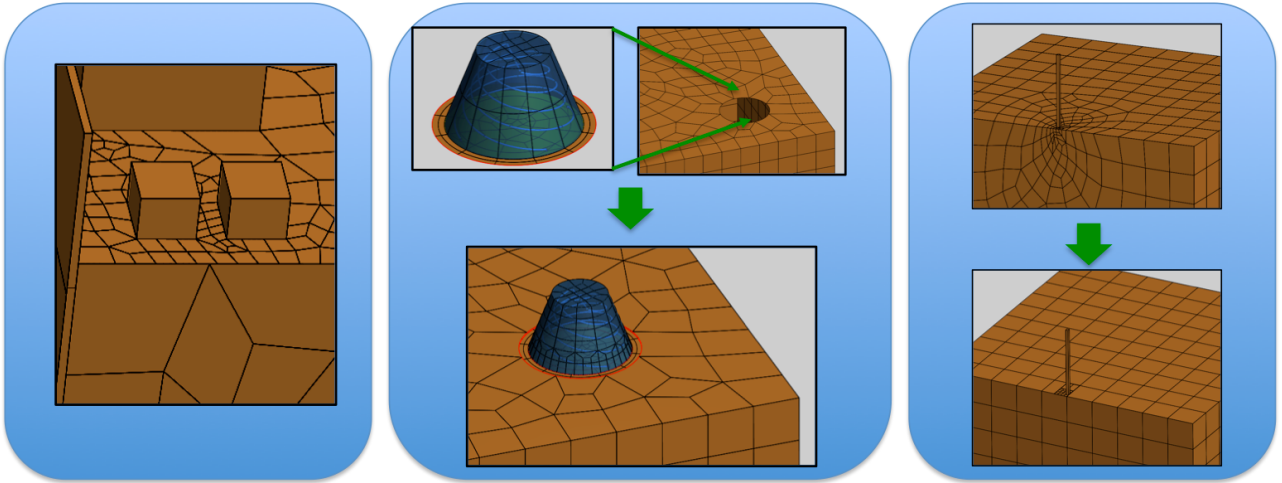


Fig. 3. Simulation-ready meshes supported by the new HO-MLFMM solver. Left: Mesh originating in CAD file with missing topological information. The current will flow seamlessly across the partially overlapping edges. Centre: Meshes originating in different sources can easily be joined, without worrying about mesh continuity. Right: Accommodation of small geometrical features in a connected mesh (top) leads to irregular meshing in a larger area than needed with a non-connected mesh (bottom).

As a further benefit, we note that a relatively large part of the required memory is occupied by the near field matrix when a high expansion order is used (typically 75 percent). With a low-order solution, the near field matrix is typically smaller (e.g. 50 percent). The relatively large near-field matrix provides two additional benefits of the HO-MLFMM:

- With a larger near field matrix, more information is available for constructing an effective preconditioner, leading to faster iterative convergence.
- The near field matrix is only used once in each iteration which implies that 75 percent of the total memory can be placed in out-of-core storage with a relatively small time penalty. The new HO-MLFMM solver also exploits this option to further extend the range of problems that can be solved with a given computer memory.

In summary, the HO-MLFMM algorithm allows for a very efficient solver using less memory and CPU time than comparable solvers.

B. Mesh robustness

The accuracy and convergence properties of an integral-equation based solver are heavily influenced by the quality of the surface mesh used as input. By far the majority of integral-equation based solvers are based on the mixed-potential EFIE that requires continuous basis functions. All commonly applied basis functions, e.g. RWGs, are continuous only when adjacent mesh elements are sharing an edge and the normal continuity of the surface current flowing across the edge is enforced. The strict continuity requirement imposed on the basis functions implies that meshes must be properly connected. If two adjacent patches only have partially overlapping edges, the continuity requirement is violated and the solution is wrong. This leads to a number of difficulties and potential errors when applying integral equation solvers to practical problems:

- 1) A mesh without proper connectivity may arise when the geometry is imported from a CAD file with missing or wrong topological information. If two faces are in physical contact but this information is not present in the CAD file, the result may be a non-connected mesh.
- 2) The connectivity requirement makes it difficult or impossible to join meshes originating from different sources. One example is when an antenna has been defined by a user-defined mesh, e.g. produced by a script or a 3rd party program, and the scattering from a platform imported from a CAD file needs to be computed. A hole can then be introduced in the platform CAD file to accommodate the antenna. However, the user-defined mesh and the platform mesh will generally not match and the result is a non-connected mesh with partially overlapping edges, and a large solution error.
- 3) A high mesh density is required when small geometrical features must be accommodated in the mesh. This high mesh density may spread to adjacent faces and the introduction of a local feature will generally require remeshing of the entire structure. The irregular region of the mesh influenced by the presence of a small geometrical feature will be larger than necessary, simply to obtain a connected mesh.

An elegant solution to the problems listed above was introduced recently [7]. By using a generalised continuous integral equation, the range of permitted basis functions can be extended such that discontinuous basis functions and non-connected meshes do not introduce solution errors. This approach was demonstrated for PEC objects and RWG basis functions in [7] but may be readily extended to the case of dielectric materials and higher-order basis functions. The new HO-MLFMM solver employs this generalized set of equations and as a consequence, there is no continuity requirement imposed on the mesh. The robustness of the solver is therefore

TABLE I
COMPUTATIONAL RESOURCES AT 1.5 GHZ.

Implementation	Processor	Memory	Time
[8]	Dual Xeon E5-2690 (Server)	170 GB	17 hours
HO-MLFMM	2.6 Ghz i7 (Laptop)	8 GB	36 min

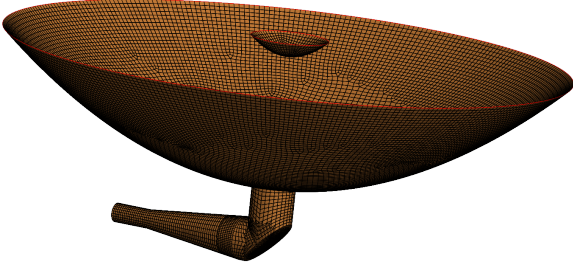


Fig. 4. Mesh for the Helios Command Station Antenna

greatly enhanced since the desired simulation accuracy is maintained on both connected and non-connected meshes. Figure 3 shows three examples of simulation-ready meshes that would not be supported by most other solvers.

C. HO-MLFMM applied to practical antenna problem

As a specific example of an actual application, we consider the scenario discussed in detail in [8], concerning the Helios Command Station antenna. This is a near-field Cassegrain configuration, utilizing a 30 m diameter main reflector along with a 4.1 m diameter subreflector and a 9.4 m long horn radiating onto a 3.2 m diameter paraboloid. The mesh of this structure using up to $2\lambda \times 2\lambda$ patches at 1.5 GHz is shown in Figure 4. The actual surface used in [8] was based on a laser scanner model while the results presented here are obtained on a nominal reflector surface. However, the electrical size and complexity of the problem are the same as in [8] which allows comparison of the computational resources used by the two implementations. The problem was solved on a laptop computer in about half an hour using 8 GB RAM. The features for out-of-core storage were not needed for this case. The computational resources are listed in Table 1 along with the resources reported in [8].

A second practical application example is shown in Figure 5. The HO-MLFMM solver is here used to compute the scattering by a large and fully detailed model of a communication satellite at Ku-band. The model includes the satellite platform itself, several antennas, as well as the solar panels. The active antenna is one of the dual-reflector antennas fed by a conical horn which is also included in the model. The problem size is equivalent to a 22 million unknown RWG problem but the solution is obtained in less than three hours. The table shown in the inset of Figure 5 lists some of the HO- MLFMM solution parameters.

III. FAST INTEGRATION OF SURFACE CURRENTS

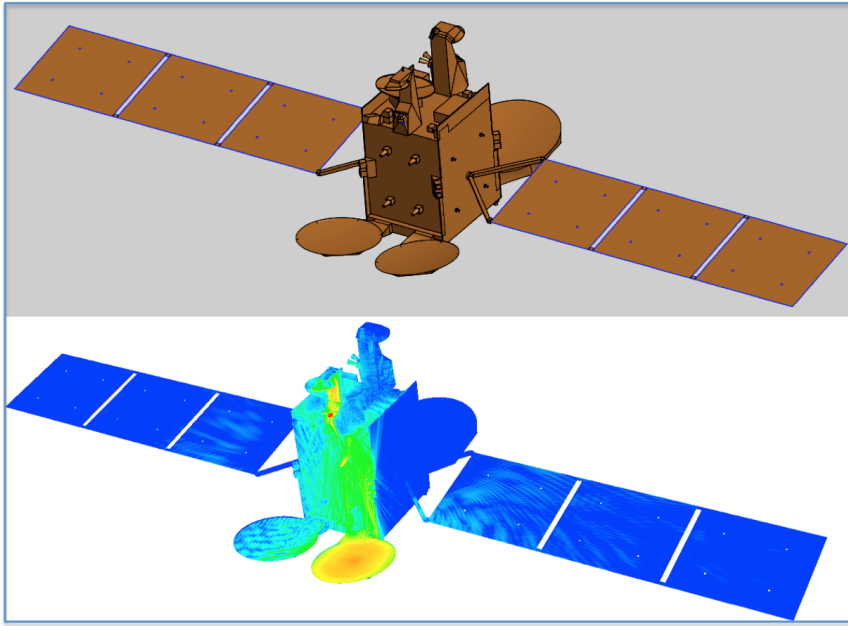
The most commonly applied methods for electrically large scattering and radiation problems are the MLFMM and the Physical Optics method. In both methods, the surface current

on a very large domain is obtained which subsequently can be used to find all other desired quantities such as the radiation pattern. The large size of the problem implies that a very large number of sample points are needed when numerically integrating the surface currents to obtain the radiated field, in particular when near fields are desired. Due to the oscillating nature of the integrand, numerical integration of the surface currents can be very time-consuming and often becomes the bottleneck. The Fast-PO algorithm, e.g. [5], has been introduced to overcome this problem. However, the existing algorithms have not demonstrated the robustness needed in industrial applications when strict accuracy requirements are present and/or when the field sample points are located in the extreme near field of the antenna. In those situations, the performance of the existing algorithms drop rapidly, due to the need for oversampling or for sub-divisioning of the problem into many smaller subproblems where the field points are no longer in the extreme near field. A new approach was introduced recently [6] and has been implemented in GRASP. This new method provides a very large computational speedup, even when the field points are located in the extreme near field.

As an example of the efficient surface current integration we choose to evaluate the extreme near field of the Helios command station antenna considered in the previous section. The surface currents need to be integrated in order to obtain the aperture field of the antenna in a plane passing $\lambda/10$ behind the sub reflector. The total number of integration points is 1.2 millions and a fairly dense output grid is needed to capture the rapidly varying near field of the antenna. We use a $\lambda/5$ sampling in each linear dimension leading to more than half a million field sample points and the details of the grid are listed in Table II. The computation time for this problem on a standard laptop computer is 86 minutes when a direct surface integration is used. However, when the new fast algorithm is applied the time drops to about 1 minute corresponding to a speedup of 82 times. The relative RMS error of the aperture field obtained with the fast method is below 10^{-4} and the result is visually identical to the aperture field obtained by a direct surface integration. The field obtained with the fast method is shown in Figure 6 where the sub reflector shadow is easy to observe as well as strong diffraction ripples originating in the reflector edges.

TABLE II
PARAMETERS OF THE NEAR FIELD COMPUTATION AT 1.5 GHZ.

Number of current samples	1.2 million
Near field grid size	30 m $\times$ 30 m
Minimum distance to subreflector	$\lambda/10$
Sample spacing	$\lambda/5$
Number of sample points	562,500
Relative error	10^{-4}
Direct surface current integration	1 h 26 m
Fast surface current integration	1 m 3 s
Computational speedup factor	82



Electrical size	$167,762 \lambda^2$
Patches	105,436
Smallest patch size	$\lambda/200$
Largest patch size	2.2λ
Basis function order	1-9
HO Unknowns	4,475,955
Equivalent RWG unknowns	22,000,000
Iterations	89
Memory	122 GB
Time	2:50 H

Fig. 5. Detailed model of communication satellite analysed at Ku-band using HO-MLFMM. Top left: CAD model (courtesy of M. Sabbadini, ESA-ESTEC). Bottom left: Induced currents on the satellite @ 12 GHz when a single reflector antenna is active. Right: HO-MLFMM solution parameters.

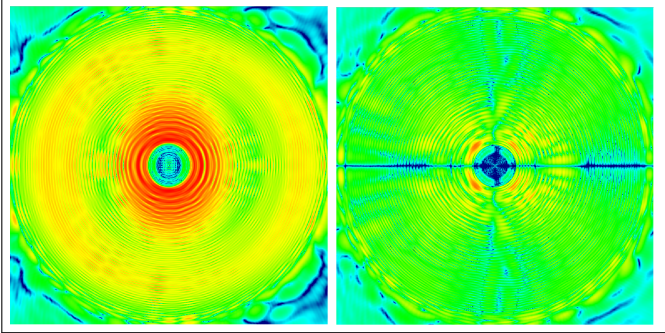


Fig. 6. Aperture field of the Helios Command Station (50 dB dynamic range), observed in the extreme near field at a plane passing $\lambda/10$ behind the sub reflector. Left: Co-polar. Right: Cross-polar.

IV. CONCLUSION

We have presented two algorithms for solving electrically large scattering and radiation problems in the frequency domain. The first algorithm is a HO-MLFMM algorithm that allows basis functions of very high order to be used in the MLFMM. The performance in terms of the memory and CPU time needed to achieve a certain accuracy was shown to be significantly better when using the new HO-MLFMM solver than when using the commonly available low-order MLFMM. In addition, the HO-MLFMM solver is based on a set of generalised integral equations that allows discontinuous basis functions and non-connected meshes to be used without loss of accuracy. This feature adds a high degree of robustness in practical applications where the mesh originates in a CAD file or in multiple independent sources. Finally, a new fast algorithm for integration of surface currents was demonstrated. It was shown that a speedup of two orders of magnitude can

be obtained even when the extreme near field and very high accuracy are required.

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