Design and Analysis of Printed Reflectarrays with Irregularly Positioned Array Elements

Min Zhou[†*], Stig Sørensen[†], Peter Meincke[†], Erik Jørgensen[†], Oleksiy S. Kim[*]
Olav Breinbjerg[*], and Giovanni Toso[‡]
[†] TICRA, Læderstræde 34, DK-1201, Copenhagen, Denmark
mz@ticra.com
[*] Department of Electrical Engineering, Electromagnetic Systems, Technical University of Denmark
Ørsteds Plads, Building 348, DK-2800 Kgs. Lyngby, Denmark
osk@elektro.dtu.dk, ob@elektro.dtu.dk
[‡] Antenna and Submilimeter Wave Section, ESA-ESTEC
Keplerlaan 1, 2200 AG Noordwijk, The Netherlands
Giovanni.Toso@esa.int

Abstract—**The design and analysis of printed reflectarrays with irregularly positioned array elements is presented. An accurate and efficient power pattern synthesis procedure, based on the Local Periodicity approach (LP) and the minimax optimization algorithm has been implemented. The analysis accuracy of the LP approach for irregular reflectarrays is established by comparisons with full wave Method of Moments solutions. It is demonstrated that the LP approach is very accurate and can be used to analyze and optimize irregular reflectarrays. Two contoured beam reflectarrays, based on a regular and an irregular layout, generating the same contoured beam pattern to cover an European region with cross-polar and sidelobe suppression have been designed. Simulations show that better performance in the cross-polar radiation can be obtained by using the irregular reflectarray, thus demonstrating the potential of reflectarrays with irregularly positioned array elements.**

I. Introduction

Printed reflectarrays are becoming viable alternatives to reflector antennas for satellite applications due to their low profile and ease of manufacture, and they are the subject of increasing research interest [1], [2]. To obtain a certain antenna performance, several degrees of freedom in printed reflectarrays can be employed, including the shape, size, orientation and position of the array elements, as well as the shape of the reflecting surface. Research on reflectarray synthesis has mostly been on exploiting the different shape, size, and orientation of the array elements [3], [4]. Recently, there have been interests in completely irregular reflectarrays where also the position of the array elements and the shape of the reflecting surface are utilized to improve the antenna performance [5]–[7]. The investigations in [5] and [6] were based on phase-only optimization that served as the starting point for the more accurate power pattern synthesis presented in [7]. Due to the strong irregularities of the designs, the analysis of each array element in [7] was performed using a full-wave Method of Moments (MoM) that included the nearest neighboring elements. As a result, the overall synthesis of the irregular reflectarray became very time consuming.

In this paper, we present the design and analysis of printed reflectarray with irregularly positioned array elements using a less time consuming synthesis procedure. Only square patches and single layer configurations will be considered in this work.

II. Analysis and Optimization Methods

A thorough investigation on accurate analysis of reflectarrays was presented in [8], where the accuracy of the analysis methods were established by comparison with measurements. The results have been used to implement an accurate and efficient power pattern synthesis procedure, where all array elements are simultaneously optimized in an iterative procedure. The analysis and optimization methods used in this synthesis are described in this section.

A. Analysis Method

The commonly adopted technique for the calculation of the electric currents on the printed array elements is based on the Local Periodicity approach (LP) where each array element is analyzed assuming that it is located in an infinite array of identical elements [9]. It was demonstrated in [8] that the LP produces accurate results, even for the cross-polar radiation, if a measured feed pattern is used to calculate the incident field, and higher order basis functions are employed to account for the singularities of the electric currents at the edges of the array elements. For reflectarrays with regularly positioned array elements, the LP is a good trade-off between accuracy and efficiency, and is therefore used in the optimization procedure presented here.

For the radiation calculations, technique II presented in [10] is used. This technique employs the field equivalence principle in conjunction with the scattering coefficients, which are calculated using the fundamental Floquet mode from the LP formulation. It yields good results in the forward hemisphere if the finite substrate and ground plane size are correctly accounted for. In addition, the scattering coefficients can be pre-calculated and stored in a look-up table, which can be accessed and interpolated during the optimization process. This greatly reduces the computation time during the optimization since the analysis of all array elements in each iteration is avoided. For a given frequency, substrate, and unit cell size, the scattering coefficients depend on two parameters: the patch dimension, and the illumination angle. The number of samples needed in the look-up table to obtain accurate interpolation for each parameter is approximately 60, and this number is independent of the array size or the number of array elements. This yields a total of 3600 scattering coefficients calculations, which can be computed within a couple of minutes. The look-up table can be reused and needs only to be recalculated if another substrate, frequency, or cell size is used.

B. Optimization Method

The optimization method is based on the minimax algorithm [11], where the maximum discrepancy between realized and specified gain in each iteration is minimized. The optimization routine is capable of optimizing for both co- and cross-polar radiation and for several feeds simultaneously, thus enabling the optimization of dual-polarized contoured/multiple beam reflectarrays including cross-polar and sidelobe suppression.

III. Irregularly Positioned Array Elements

To utilize the position of the array elements in the optimization, an irregular distribution of element positions is obtained through a mapping from a regular to an irregular grid. Appropriate constraints on the mapping must be enforced to ensure the area of the reflectarray surface is efficiently utilized. In contrast to [7], where the edges of the reflectarray are not constrained, the edges are kept fixed in our case to avoid any undesired increase in antenna size introduced by the mapping.

In the present work, the mapping is obtained by adding a distortion to the regular grid. Define (α, β) as normalized positions in the regular grid such $|\alpha| \leq 1$ and $|\beta| \leq 1$. Then, the new normalized position in the irregular grid is given by $(\alpha', \beta') = (\alpha + f_x, \beta + f_y)$, where

$$f_x(\alpha, \beta) = (\alpha - 1)(\alpha + 1) \sum_{p=0}^{P} \sum_{q=0}^{Q} c_{pq} T_p(\alpha) T_q(\beta). \quad (1)$$

Herein, T_i is the Chebyshev polynomial of order i, and c_{pq} the weighting coefficients. The terms in front of the summations are to ensure that the edges of the reflectarray are kept fixed. The function f_y is of the same form with another set of weighting coefficients.

The degree of the distortion is determined by the value of the weighting coefficients and the order i. To avoid too high degree of distortions, where array elements may overlap, upper and lower bounds must be specified for the weighting coefficients, and the maximum order of the Chebyshev polynomials should not exceed 4. Only a few, 2-6, weighting coefficients are needed to achieve strong irregularities, as will be shown in Section IV. The weighting coefficients are optimized at each iteration together with the dimension of the array elements.

IV. Reflectarray Analysis

The use of the LP for the design and analysis of reflectarrays with irregularly positioned array elements is new, and thus the accuracy of the LP for such reflectarrays has to be established. This issue is essential since an accurate analysis is required to obtain an optimal design. In this section, the use of the LP for the analysis of reflectarray with irregularly positioned array elements will be described, and the accuracy will be evaluated.

A. The Analysis of the Distorted Cell

Due to the grid distortion, the unit cells in the distorted grid are not rectangular but curved. The center of the array element within the distorted cell is positioned at the center of the distorted cell, which is defined to be at the intersection of the two diagonal lines of the distorted cell. To ensure the best usage of the area within the distorted cell, the array element is rotated to orient in parallel of the bisector lines of the two diagonal lines of the distorted cell. This is illustrated in Fig. 1a, where the diagonal and bisector lines are shown as solid and dashed lines, respectively. Such a distorted cell can not be used in the LP approach since the implemented algorithm [12] assumes unit cells that are either rectangles or parallelograms. Thus, an equivalent square cell with the same area as the distorted cell is defined. The equivalent cell has the same center as the distorted cell and is oriented in parallel of the bisector lines of the two diagonal lines of the distorted cell. This is shown in Fig. 1b. The equivalent cell is used in the LP computations to calculate the scattering coefficients. Equivalent cells that are rectangles or parallelograms may be better choices, but numerical results, which will be presented in Section IV-B, show that square cells are adequate.

The analysis procedure for reflectarrays with irregularly positioned elements is then the same as for those with regularly positioned elements, except that the unit cells of the array elements are of different sizes. This affects the scattering coefficients and a modification of the look-up table, which was mentioned in Section II-A, is required. In addition to the storage of patch dimensions and illumination angles, samples of different sized unit cells have to be stored as well. The number of unit cell samples needed depends on the degree of the grid distortion. For the results presented in this paper, approximately 50 samples were sufficient, yielding a look-up table that requires $3600 \times 50 = 180000$ scattering coefficients calculations per frequency. The computation time on a 2.8 GHz Intel processor laptop is approximately one hour which is a significant increase compared to the regular array case. It is, however, still acceptable since the look-up table only needs to be calculated once.

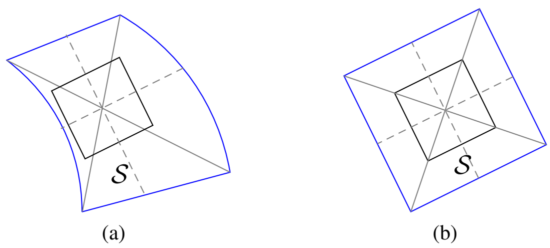


Fig. 1. An example of (a) a distorted cell and (b) its equivalent square cell. Both cells have the area $\mathcal{S}$. The center of the patch is located at the intersection of the two solid diagonal lines of the distorted cell. The rotation of the patch is given by the dashed bisector lines.

B. The Analysis Accuracy

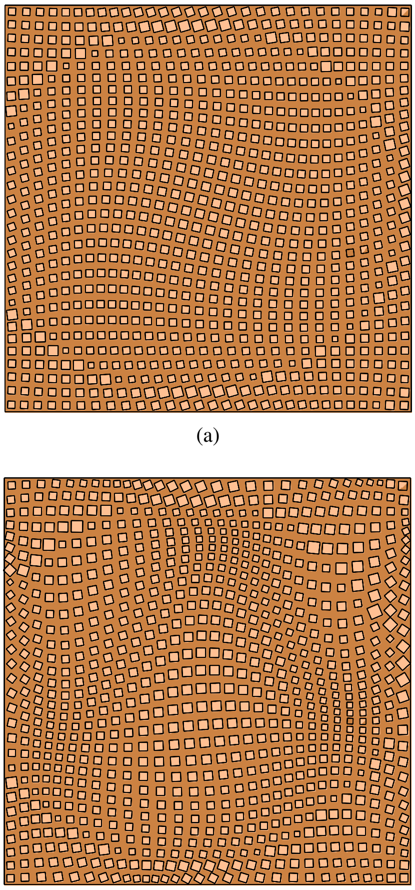
To evaluate the analysis accuracy of the LP for reflectarrays with irregularly positioned array elements, two irregular offset pencil beam reflectarrays are designed. Two different distortions are selected, a moderately distorted and a highly distorted grid. The distorted grids are kept fixed, and the array elements are optimized to radiate a pencil beam towards the specular direction. The masks of the optimized reflectarrays are shown in Fig. 2. The number of weighting coefficients for the moderately and highly distorted grids is 2 and 6, respectively. The geometrical parameters are summarized in Table I. The feed is a vertically polarized corrugated horn with a taper of $-17.5\,\mathrm{dB}$ at $30°$ at $10\,\mathrm{GHz}$. The feed has been measured at the DTU-ESA Spherical Near-Field Antenna Test Facility [13], and its measured feed pattern is used in the optimization.

To serve as reference, a full wave MoM is used. To calculate the electric currents on the array elements, the MoM uses the spatial dyadic Greens function for grounded dielectric slab [14], thus it assumes infinite substrate and ground plane. For the radiation calculations, the finite substrate and ground plane size is accounted for by using technique III from [10].

The radiation patterns of the two reflectarrays calculated using the LP and MoM are shown in Fig. 3. An excellent agreement between the two methods is observed for the moderately distorted case in Fig. 3a. Here, the co-polar patterns of LP and MoM practically coincide. The agreement for the highly distorted case is slightly worse compared to the moderate case. This is seen in Fig. 3b, where some discrepancies are observed. Nevertheless, the LP is capable of predicting most of the features that is predicted by MoM. Regarding the cross-polar radiation, there are some discrepancies between the LP and MoM. However, the maximum cross-polar radiation predicted by both methods are close and the deviation is around $1-3\,\mathrm{dB}$. The highly distorted case is not a realistic case as the distortion deteriorates the performance of the antenna.

The good accuracy of the LP is somewhat surprising since the irregularities of the reflectarrays are rather strong, but must be attributed to the relatively smooth patch variation and the systematic manner in which the grids are distorted. Several irregular designs have been designed and analyzed using both LP and MoM, and it was seen that the accuracy of LP is generally good.

This investigation show that the LP is accurate for any realistic grid distortion and can be used to analyze and optimize reflectarrays with irregularly positioned array elements.



(a)

(b)

Fig. 2. Irregular reflectarrays with (a) a moderately and (b) a highly distorted grid. The reflectarrays are designed to radiate a pencil beam towards the specular direction.

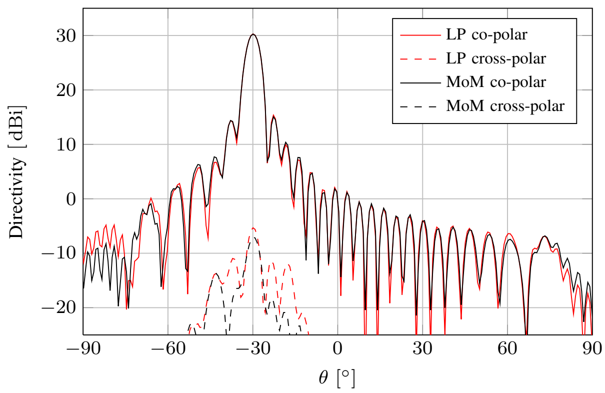
V. Reflectarray Design

We consider an offset contoured beam reflectarray that radiate a high-gain beam within an European coverage with cross-polar suppression within the same coverage and sidelobe suppression within a southern African contour. The coverages are shown in Fig. 4 as red polygons. The reflectarray consists of 2500 array elements and is designed for a single linear polarization. The dimension of the reflectarray is selected to $60 \times 60\,\mathrm{cm}^2$ corresponding to approximately 20×20 square wavelengths at $10\,\mathrm{GHz}$. The corrugated horn used in the previous pencil beam designs is used as a feed, and its measured pattern is used in the optimization. The grid distortion is no longer kept fixed and ten weighting coefficients are included in the optimization. The geometrical parameters are summarized in Table II.

First, a reflectarray with regularly positioned elements to cover only the European region without any constraints on the cross-polar and sidelobe level is optimized (Design I). Identical patches are used as the initial guess for the array elements. In this design, the minimum cross polarization discrimination (XPD) is around $22.6\,\mathrm{dB}$, and the minimum high/low (Europea/Africa) gain isolation is $15.6\,\mathrm{dB}$.

TABLE I
PENCIL BEAM REFLECTARRAY DATA

Frequency	$10\,\mathrm{GHz}$
Number of elements	30×30
Reflectarray dimensions	$405\,\mathrm{mm} \times 405\,\mathrm{mm}$
Substrate thickness	$0.762\,\mathrm{mm}$
Relative permittivity (ϵ_r)	3.66
Loss tangent ($\tan\delta$)	0.0037
Feed distance to center of array	$0.60\,\mathrm{m}$
Feed offset angle	$\theta = 30°$, $\phi = 0°$
Main beam direction	$\theta = -30°$, $\phi = 0°$



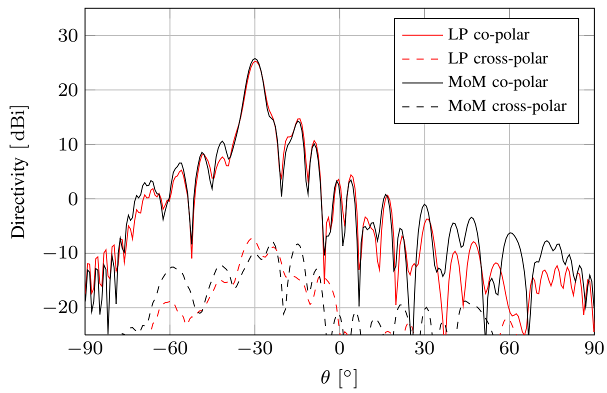


Fig. 3. Comparison of the radiation pattern calculated using LP and MoM for (a) the moderately distorted case and (b) the highly distorted case.

TABLE II
CONTOURED BEAM REFLECTARRAY DATA

Frequency	10 GHz
Number of elements	50×50
Reflectarray dimensions	600 mm × 600 mm
Substrate thickness	0.762 mm
Relative permittivity (ϵ_r)	3.66
Loss tangent ($\tan\delta$)	0.0037
Feed distance to center of array	0.60 m
Feed offset angle	$\theta = 30°$, $\phi = 0°$
Main coverage	European coverage
Cross-polar suppression	European coverage
Sidelobe suppression	Southern African coverage

Cross-polar within the European coverage and sidelobe suppression within the southern African coverage are subsequently added in the optimization goals, and two reflectarrays are designed, one with a regular grid (Design II), and another one with an irregular grid (Design III). For brevity, only the mask and radiation of design III is shown. The co- and cross-polar radiation patterns are shown in Fig. 4 and the mask of the antenna in Fig. 5. A summary of the performance of the designs is listed in Table III.

Improvements in minimum XPD and isolation level are observed for both designs II and III. However, these improvements are obtained at the cost of a reduction of 0.5 dB in the minimum directivity within the European coverage. The improvements in the minimum XPD for design III is interesting. It indicates that a better performance in the cross-polar radiation can be obtained by using an irregular array instead of a regular one. Even though the improvements are small, the present research on irregular reflectarrays is work in progress and further investigations should be conducted. Due to the large electrical size of the antennas, a full wave MoM was not possible and reference solutions of these designs do not exist. However, design III is being manufactured and comparison of numerical simulations and measurements will soon be available to validate the design.

The improvements obtained in the XPD for antenna III was obtained for a single polarization. For contoured beam applications, dual-polarization is often of interest, thus the performance of an irregular array for dual-polarization should be investigated. To this end, regular and irregular reflectarrays optimized for two orthogonal linear polarizations have been designed. Good results for both polarizations can be obtained with a regular array, but no further improvement is gained by using an irregular array. This conclusion is preliminary and many possibilities can still be investigated to improve the antenna performance, e.g. other types of distortions.

The reflectarrays presented in this paper are based on square patches. It is expected that further improvements can be achieved by adding more degrees of freedom in the optimization, e.g. using rectangular patches or other types of array elements, etc.

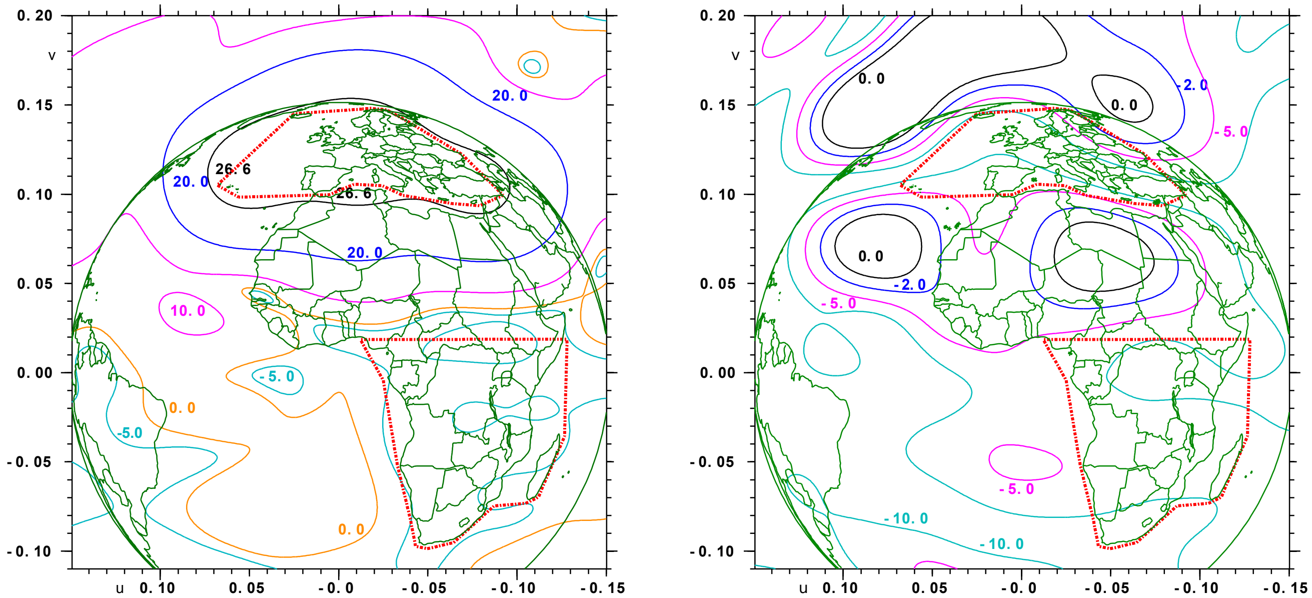
TABLE III
PERFORMANCE OF REGULAR VERSUS IRREGULAR GRID WITH AND WITHOUT CROSS-POLAR AND SIDELOBE SUPPRESSION

	Min. directivity	Min. XPD	Min. isolation
Design I	27.1 dBi	22.6 dB	15.6 dB
Design II	26.6 dBi	27.5 dB	27.8 dB
Design III	26.6 dBi	28.3 dB	27.6 dB

VI. CONCLUSION

The analysis and design of reflectarrays with irregularly positioned array elements have been presented. An accurate and efficient power pattern synthesis procedure, based on the Local Periodicity approach and the minimax algorithm has been implemented and is used in the design of irregular reflectarrays. The analysis procedure has been described and the accuracy of the Local Periodicity approach for irregular reflectarrays is established by comparisons with full wave method of moments solutions. It is shown that the Local Periodicity approach is surprisingly accurate and can be used to analyze and optimize irregular reflectarray.

Two offset contoured beam reflectarrays, based on a regular and an irregular layout, generating the same contoured beam pattern to cover an European coverage with cross-polar suppression within the same coverage and sidelobe suppression within a southern African contour have been designed. The reflectarrays are optimized for a single feed polarization. Simulations indicate that an enhanced performance in the cross-polar radiation can be obtained using an irregular array



(a) (b)

Fig. 4. The (a) co-polar and (b) cross-polar radiation of design III at 10 GHz. The coverages are defined by the red polygons. The reflectarray is optimized for high-gain radiation with cross-polar suppression within the European coverage and sidelobe suppression within the southern African coverage.

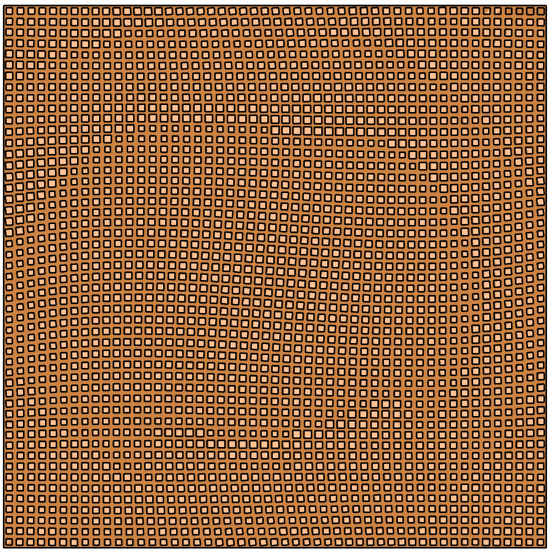


Fig. 5. The mask of design III.

instead of a regular array. The irregular array is currently being manufactured and comparison of simulations and measurements will be presented at the conference to validate the design. Investigations for dual-polarization applications have also been conducted, and results show that no improvements are gained by the irregular reflectarray. However, this conclusion is preliminary and further investigations on this matter must be conducted.

Acknowledgment

Dr. S. Pivnenko, Technical University of Denmark, is acknowledged for the measurements of the corrugated horn.

References

[1] J. Huang and J. A. Encinar, *Reflectarray Antennas*. IEEE Press, 2008.
[2] J. A. Encinar, "Recent advances in reflectarray antennas," in *Proc. EuCAP*, Barcelona, Spain, 2010.
[3] D. M. Pozar and T. A. Metzler, "Analysis of a reflectarray antenna using microstrip patches of variable size," *Electron. Lett.*, vol. 29, no. 8, pp. 657–658, 1993.
[4] J. Huang and R. J. Pogorzelski, "A Ka-band microstrip reflectarray with elements having variable rotation angles," *IEEE Trans. Antennas Propag.*, vol. 46, no. 5, pp. 650–656, 1998.
[5] A. Capozzoli, C. Curcio, E. Iavazzo, A. Liseno, M. Migliorelli, and G. Toso, "Phase-only synthesis of a-periodic reflectarrays," in *Proc. EuCAP*, Rome, Italy, 2011, pp. 987 – 991.
[6] A. Capozzoli, C. Curcio, A. Liseno, M. Migliorelli, and G. Toso, "Aperiodic conformal reflectarrays," in *Proc. IEEE AP-S Int. Symp.*, Spokane, Washington, USA, 2011, pp. 361 – 364.
[7] ——, "Power pattern synthesis of advanced flat aperiodic reflectarrays," in *Proc. 33th ESA Antenna Workshop*, Noordwijk, The Netherlands, 2011.
[8] M. Zhou, S. B. Sørensen, O. S. Kim, S. Pivnenko, and G. Toso, "Investigations on accurate analysis of microstrip reflectarrays," in *Proc. 33th ESA Antenna Workshop*, Noordwijk, The Netherlands, 2011.
[9] D. M. Pozar, S. D. Targonski, and H. D. Syrigos, "Design of millimeter wave microstrip reflectarrays," *IEEE Trans. Antennas Propag.*, vol. 45, no. 2, pp. 287–296, 1997.
[10] M. Zhou, S. B. Sørensen, E. Jørgensen, P. Meincke, O. S. Kim, and O. Breinbjerg, "An accurate technique for calculation of radiation from printed reflectarrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 1081–1084, 2011.
[11] J. Hald and K. Madsen, "Combined LP and Quasi-Newton methods for minimax optimization," *Math. Programming*, vol. 20, pp. 49–62, 1981.
[12] G. Kristensson, S. Poulsen, and S. Rikte, "Propagators and scattering of electromagnetic waves in planar bianisotropic slabs - an application to frequency selective structures," *Progr. Electromagn. Res. (PIER)*, vol. 48, pp. 1–25, 2004.
[13] "DTU-ESA Spherical Near-Field Antenna Test Facility," *http://www.dtu.dk/centre/ems/English/research/facilities.aspx*.
[14] Y. Chow, J. Yang, D. Fang, and G. Howard, "A closed-form spatial Green's function for the thick microstrip substrate," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 3, pp. 588 – 592, 1991.