

# Analysis and Optimization of a Curved Transmit-Receive Contoured Beam Reflectarray

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**Abstract**—This paper presents the analysis and optimization of a 1 meter single-layer curved contoured beam reflectarray in Ku-band. The curved reflectarray is designed to radiate a contoured beam over a European coverage in both transmit and receive frequency bands for dual linear polarization. For the analysis of the curved reflectarray, the spectral domain method of moments assuming local periodicity is used, and its accuracy is verified by comparisons with full-wave method of moments simulations. For the optimization, a direct optimization approach where all the array elements are simultaneously optimized is used. The optimized curved reflectarray is compared to the shaped reflector, and the comparison shows that the performance of the curved reflectarray is identical to that of the shaped reflector.

**Index Terms**—reflectarrays, contoured beam, optimization, shaped reflectors, satellite applications

## I. INTRODUCTION

Printed reflectarrays usually consist of a flat surface. They are light, easy and cheap to manufacture, and provide a way to realize low-cost high-gain antennas for space applications [1]. For satellite broadcasting applications, where contoured beams that radiate certain geographical areas in a large frequency band are required, the shaped reflector is the preferred choice for most missions. Although the shaped reflector is based on a mature technology and has proven to be a reliable solution, the cost associated to its manufacturing is high. Consequently, satellite manufacturers and space agencies are investigating possible cheaper solutions in the form of reflectarrays.

Contoured beam reflectarrays have been reported in various works, e.g., [2]–[4]. However, the bandwidth of the reflectarrays presented in these works is only sufficient for either transmit (Tx) or receive (Rx) operation. In [5], a reflectarray operating in both Tx and Rx frequency bands was presented, the simulations show that the coverage gain requirements are fulfilled in more than 90% of the region in both frequency bands. A common feature of the reflectarrays presented in [2]–[5] is that they are designed using a phase-only optimization approach, which may result in sub-optimal designs since intermediate steps are required in the design process.

A direct optimization approach where all the array elements are simultaneously optimized tends to produce improved designs. Such an approach was presented in [6]. Using the direct optimization approach, a single-layer planar contoured beam reflectarray, optimized for the same coverage requirements as in [5], was presented in [7]. This reflectarray fulfills all the coverage requirements in both Tx and Rx frequency bands and shows significantly better performance than previously

reported planar contoured beam reflectarrays. Despite exhibiting enhanced performance, this reflectarray did not reach the performance of the shaped reflector, due to the bandwidth limitations imposed by the differential spatial phase delay from the feed horn.

Several solutions have been proposed in the literature to alleviate the differential spatial phase delay, one of them being the use of advanced broadband elements in conjunction with a multi-faceted structure as suggested in, e.g., [8] and demonstrated in [4]. This solution is rather promising and it was shown in [9] that the combination of the multi-faceted concept and the direct optimization technique can yield reflectarrays with performances that are close to that of the shaped reflector. However, a reflectarray with elements printed on a doubly curved surface has a number of distinct advantages compared to its multi-faceted counterpart.

First, for a curved reflectarray, the spatial phase delay issue is further reduced compared to a multi-faceted reflectarray and the bandwidth is mainly determined by the bandwidth of the individual array elements. Second, a large doubly curved surface is inherently stiffer and can thus be made more lightweight than the corresponding multi-faceted or planar counterpart. Third, a curved reflectarray avoids the abrupt surface change at the joints of a multi-faceted reflectarray which can give rise to undesired RF effects. Finally, a curved reflectarray is attractive compared to traditional shaped reflectors due to the possibility of reusing a standard parabolic mould for multiple coverages and missions.

The concept of a curved reflectarray was first suggested in [10] and later investigated in [11]. In the latter, a two-layer reflectarray consisting of varying-sized patches on a parabolic surface is presented. Although it was shown that the bandwidth was improved, the reflectarray was synthesized using a phase-only optimization technique and the analysis of the reflectarray is carried out with some rather crude geometrical approximations, which do not take into account the curvature of the reflectarray.

In this work, we apply the direct optimization technique from [6] to design and optimize a curved reflectarray to radiate a contoured beam in both Tx- and Rx frequency bands. The reflectarray consists of single-layer elements of variable size printed on a doubly curved surface and simulations show that the performance of the curved reflectarray is identical to that of the shaped reflector.

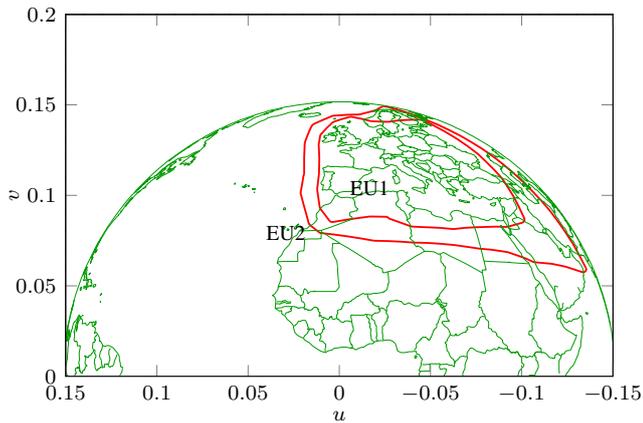


Fig. 1. The European coverages seen from the longitude 5° W geostationary orbital position.

## II. COVERAGE REQUIREMENTS

In [3], a reflectarray was designed to produce a contoured beam for a European coverage in horizontal (H) polarization and a pencil beam for a North American coverage in vertical (V) polarization in the frequency band 11.45-12.75 GHz. The coverage requirements selected in this work for the design of the curved reflectarray is a modification of the requirements from [3]. The curved reflectarray should produce a contoured beam on the European coverage for dual linear polarization, H- and V-polarization, in both Tx (11.45-12.75 GHz) and Rx (13.75-14.25 GHz) frequency bands. The coverages are shown in Fig. 1 and the co-polar directivity ( $D$ ) and cross-polar discrimination (XPD) requirements are summarized in Table I.

TABLE I  
CO- AND CROSS-POL. REQUIREMENTS

| Zone | Tx: 11.45 – 12.75 GHz |                       | Rx: 13.75 – 14.25 GHz |                       |
|------|-----------------------|-----------------------|-----------------------|-----------------------|
|      | $D_{\min}$<br>[dBi]   | XPD $_{\min}$<br>[dB] | $D_{\min}$<br>[dBi]   | XPD $_{\min}$<br>[dB] |
| EU1  | 28.5                  | 30                    | 28.5                  | 30                    |
| EU2  | 25.5                  | 30                    | 25.5                  | 30                    |

## III. REFLECTARRAY DEFINITION

In this work, we consider a single-offset configuration. The reflectarray is elliptical with major and minor half axes of 0.525 m and 0.5 m, respectively, which corresponds to a maximum diameter of approximately  $48 \lambda_0$ , with  $\lambda_0$  being the free-space wavelength, at the highest frequency.

The feed is modelled by a linearly polarized Gaussian beam with a taper of -18 dB at  $26^\circ$  and is located 1 m from the reflectarray center, yielding a focal to diameter ratio of  $F/D = 1$ .

As array element, the rectangular loop/patch combination element, which was also used in [7], is employed. The substrate has a thickness of  $h = 4$  mm with a dielectric constant of  $\epsilon_r = 1.05$  and a loss tangent of  $\tan \delta = 0.00083$ . The size of the unit cell is  $13.33 \text{ mm} \times 13.33 \text{ mm}$ .

Regarding the curvature of the reflectarray, a paraboloidal surface is used. However, the focal length ( $f$ ) of this surface must be selected with care. For a pencil beam design, the focal length should be selected such that one gets a focused configuration like a parabolic reflector, see Fig. 2a. For a contoured beam design on the other hand, a focused configuration may be a bad candidate, the reason being that the radiation from this surface (without the presence of the array elements) will be a focused beam of a size determined by the antenna aperture. If the desired coverage area is much larger than the beam width, the array elements need to compensate for the narrow beam and this can be challenging in a wide bandwidth. Thus, a better solution is to use a slightly defocused configuration, see Fig. 2b, to obtain an initial beam that is more similar to the specified coverage area. In this way, less constraints are put upon the array elements and better designs can be obtained. For the specific case considered in this paper, the focal length of the paraboloidal surface is selected to be  $f = 1.7$  m.

## IV. REFLECTARRAY ANALYSIS AND OPTIMIZATION

The direct optimization technique from [6] has been extended to allow the optimization of curved reflectarrays. The analysis method used in the direct optimization technique is

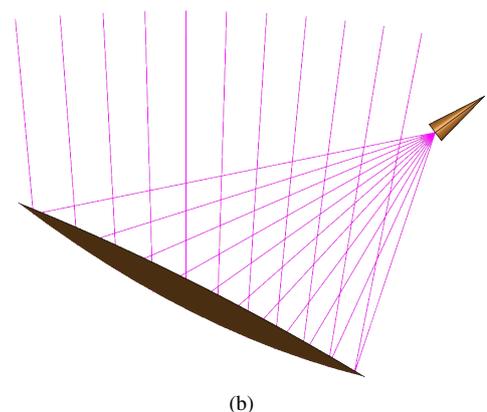
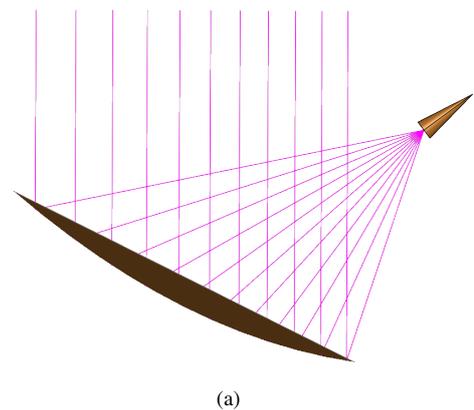


Fig. 2. The geometry of (a) focused configuration and (b) defocused configuration.

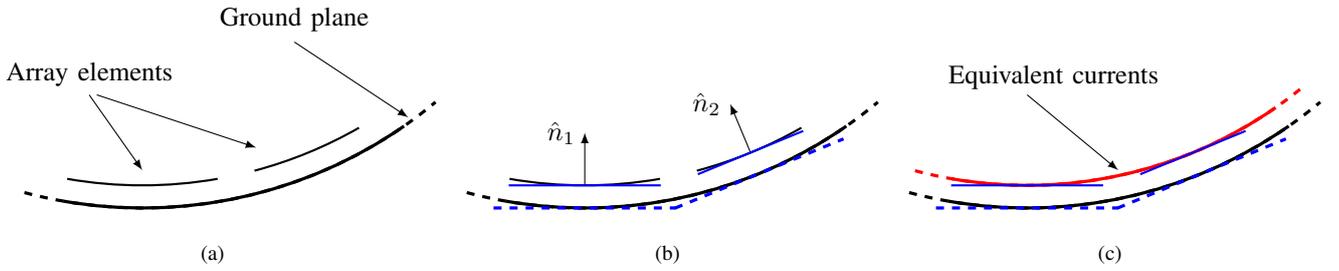


Fig. 3. Description of how the LP-SDMoM is applied on curved reflectarrays: (a) the actual reflectarray configuration of interest and (b) its equivalent configuration used in the LP-SDMoM where the array elements are assumed to be locally planar and the normal vectors  $\hat{n}$  are used to determine the angle of incidence, (c) finally based on the solution from the LP-SDMoM, equivalent currents on the top surface of the curved reflectarray are defined from which the far-field is calculated.

based on a spectral domain method of moments (SDMoM) assuming local periodicity (LP). The optimization engine uses a gradient-based non-linear minimax optimization algorithm, which is the same algorithm used in TICRA's software POS [12], which is considered by the antenna community to be the de-facto standard software for the design of shaped reflectors.

The use of the LP-SDMoM for the analysis of curved reflectarrays is new and will therefore be briefly discussed in the following. For additional details on the optimization in the direct optimization technique, the reader is referred to [6].

The configuration under consideration is shown in Fig. 3a. Here, the array elements, which are curved, are printed on a curved substrate backed by a metallic ground plane. Due to the curvature of the array elements and the ground plane, the LP-SDMoM can not be directly applied. Therefore, an equivalent planar configuration has to be defined to approximate locally the curvature of each array element. This is shown in Fig. 3b where each array element is assumed to be locally planar and shown with solid blue. Each element is then analyzed using the LP-SDMoM from which the scattering matrix for each array element is obtained. The angle of incidence in the LP-SDMoM computations is determined by the normal vector  $\hat{n}$  of each array element as shown in Fig. 3b and the size of the unit cell is highlighted using dashed blue lines.

To calculate the far-field, the Floquet harmonics technique [13, Technique II] is used. This method is based on the field equivalence principle and uses the scattering matrix of each array element to calculate the equivalent currents. The equivalent currents, which are shown in Fig. 3c with red, are defined on the top surface of the reflectarray and thereby takes into account the curvature of the reflectarray. From these equivalent currents, the far-field of the curved reflectarray is determined.

As will be shown in Section VII, this approach is exceptionally accurate for curved reflectarrays.

## V. REFLECTARRAY DESIGN

The design of the curved reflectarray is done in two steps and is similar to that presented in [7].

First, a phase-only optimization is used to design a reflectarray that partially fulfills the coverage requirements. To this end, the POS software is used to design a shaped reflector

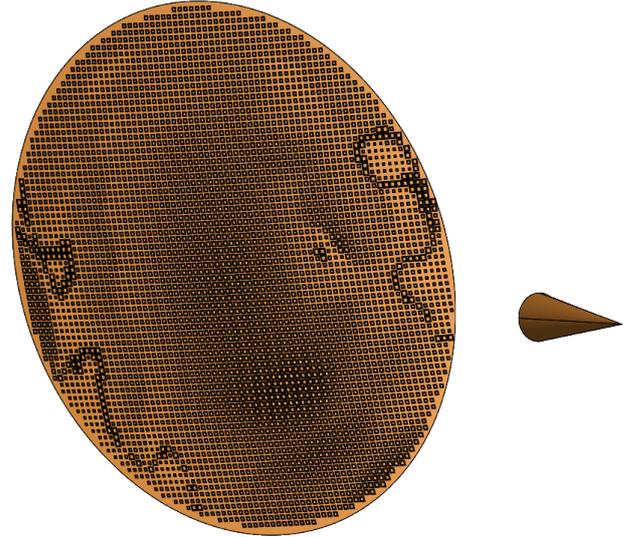


Fig. 4. The geometry of the optimized curved reflectarray.

that fulfills the coverage requirements. From the shaped reflector, the required phase distributions at the surface of the reflectarray is extracted, and the array elements are optimized, element by element, to match these phase distributions.

This phase-only design is subsequently used as the starting point for the direct optimization from which the final design is obtained. The optimization is performed at the centre and extreme frequencies of the Tx and Rx band, namely 11.45, 12.00, 12.75, 13.75, 14.00, and 14.25 GHz. The final optimized reflectarray is shown in Fig. 4.

## VI. REFLECTARRAY PERFORMANCE

The performance of the curved reflectarray has been evaluated and the simulations show that the reflectarray fulfills all the coverage requirements for both polarizations in both the Tx and Rx frequency bands. In Fig. 5, the radiation patterns, directivity and XPD, of the curved reflectarray at 11.45 and 14.25 GHz for V-polarization are shown. It is seen that the reflectarray radiates a contoured beam within EU1 and EU2 with a minimum directivity of  $D_{\min} = 29.2$  dBi and  $D_{\min} = 26.2$  dBi, respectively, which is 0.7 dB above the specifications. It is also observed that the minimum XPD in

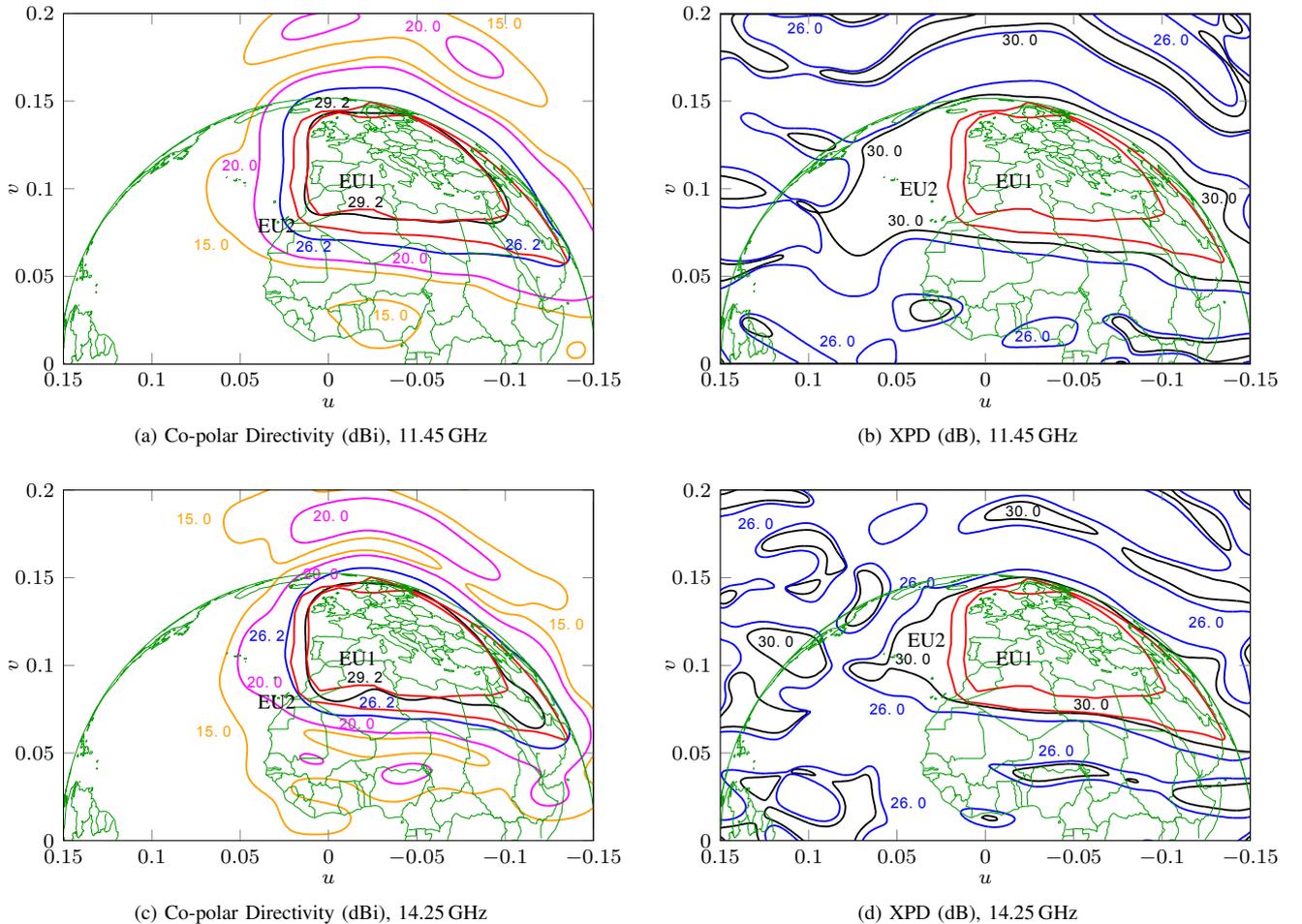


Fig. 5. The radiation pattern in V-polarization for the optimized reflectarray at 11.45 and 14.25 GHz.

the european coverage is above 30 dB. The performance in H-polarization is practically identical and is therefore not shown.

The performance of the curved reflectarray is summarized in Table II and compared to that of an equivalent sized shaped reflector optimized using POS with the same coverage requirements. It is seen that the performance of the curved reflectarray is practically identical to that of the shaped reflector.

It is, to our knowledge, the first time that the performance of a reflectarray is shown to be identical to that of a shaped reflector and this can be considered a practical breakthrough. The good performance is attributed to the combination of the doubly curved structure, a suitable reflectarray element, and the direct optimization technique used to design the reflectarray. It is our belief that the inclusion of additional degrees of freedom in the array elements and the use of a more sophisticated starting point and optimization procedure, reflectarrays that surpass the performance of the shaped reflector can be realized. This is subject to future investigation.

## VII. VALIDATION BY FULL-WAVE SIMULATIONS

To validate the results presented above, we resort to full-wave simulations since measurements are not available.

TABLE II  
COMPARISON BETWEEN REFLECTARRAY AND SHAPED REFLECTOR

| Zone | Tx: 11.45-12.75 GHz |                  |                  |                  | Rx: 13.75-14.25 GHz |                  |                  |                  |
|------|---------------------|------------------|------------------|------------------|---------------------|------------------|------------------|------------------|
|      | Curved Reflectarray |                  | Shaped Reflector |                  | Curved Reflectarray |                  | Shaped Reflector |                  |
|      | $D_{\min}$ [dBi]    | $XP_{\min}$ [dB] | $D_{\min}$ [dBi] | $XP_{\min}$ [dB] | $D_{\min}$ [dBi]    | $XP_{\min}$ [dB] | $D_{\min}$ [dBi] | $XP_{\min}$ [dB] |
| EU1  | 29.2                | 30.7             | 29.2             | 32.0             | 29.2                | 31.3             | 29.2             | 31.9             |
| EU2  | 26.2                | 30.7             | 26.2             | 30.7             | 26.2                | 30.7             | 26.2             | 30.7             |

For the present case, the dielectric constant of the substrate is  $\epsilon_r = 1.05$ , i.e., close to that of free-space. Thus the substrate can be removed without significantly affecting the performance of the reflectarray. However, even in this case, due to the size of the reflectarray, the number of unknowns required to achieve accurate results using a higher order (HO) Method of Moments (MoM) [14] at 14.25 GHz is approximately 1.3 millions, resulting in a memory consumption of approximately 6.2 TB. This is not computationally affordable, thus we resort to a multilevel fast multipole method (MLFMM) algorithm, which was recently presented in [15]. This implementation uses a HO discretization and reduces the memory requirements

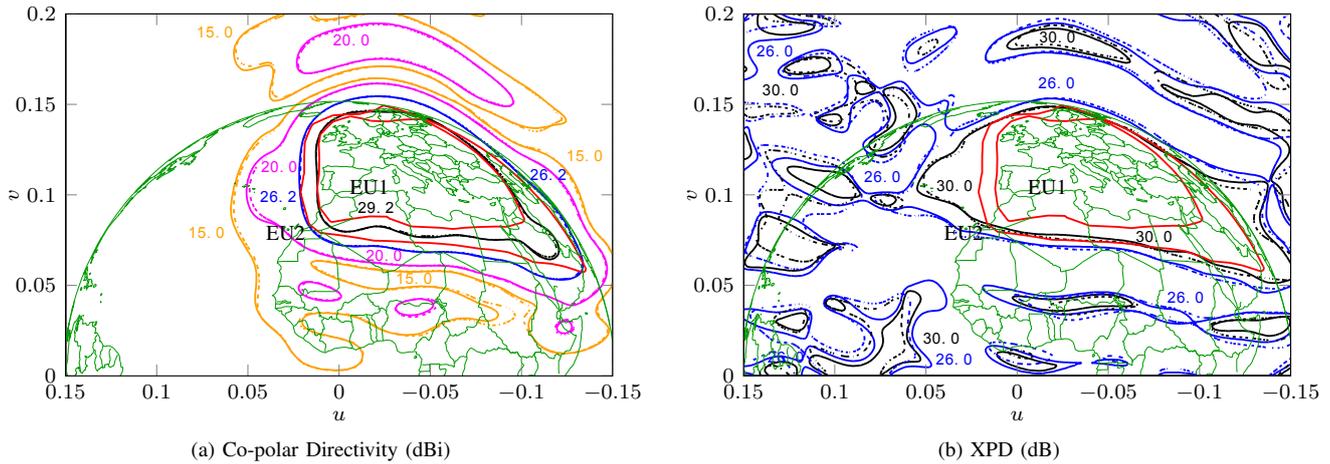


Fig. 6. Radiation patterns calculated using HO-MLFMM (solid) and LP-SDMoM (dashed) of the optimized reflectarray (without the presence of the dielectric substrate) at 14.25 GHz in V-polarization.

significantly, without loss of accuracy, hence enabling full-wave analysis of the entire reflectarray.

Using the HO-MLFMM and LP-SDMoM, the curved reflectarray has been analyzed, without the presence of the dielectric substrate, and the radiation pattern at 14.25 GHz for V-polarization is shown in Fig. 6. It is seen that an extremely good agreement between the two methods is obtained. Even for the cross-polarization, which is more than 30 dB below peak, the radiation is well predicted by the LP-SDMoM. The radiation patterns shown in Fig. 6 differ slightly with that in Fig. 5 due to the different dielectric constant used in the simulations.

The good agreement between the two methods verifies the accuracy of the LP-SDMoM for curved reflectarrays and thus also the results presented in Section VI.

### VIII. CONCLUSIONS

This paper presents the analysis and optimization of a curved contoured beam reflectarray in Ku-band. The curved reflectarray is optimized to radiate a contoured beam over a European coverage in both transmit and receive frequency bands for dual linear polarization. The reflectarray is optimized using a direct optimization technique where all the array elements are optimized simultaneously. The optimized reflectarray fulfills all the coverage requirements for both polarizations in both Tx and Rx frequency bands and its performance is identical to what is achievable for a shaped reflector. This is the first time that a reflectarray with identical performance of that of a shaped reflector is presented.

The work presented in this paper demonstrates that a reflectarray is capable of reaching, and maybe even surpassing, the performance of a shaped reflector. It also shows that reflectarrays are viable candidates for replacing shaped reflectors for satellite telecommunication and broadcasting applications.

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