Efficient Surface Optimization of Large Dual Reflector Systems

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***Abstract*—Surface shaping of reflector antennas for creating a contoured beam is an ubiquitous task in the design of modern communication satellites. In this paper, we highlight two major improvements to the state-of-the-art that provides a major increase in computational efficiency — a modern minimax optimisation algorithm for large-scale problems and the implementation of analytical derivatives for simultaneous shaping of the surface of both reflectors in a dual reflector system.**

I. Introduction

Many modern communication satellites require contoured beams in order to provide coverage to a particular region of the Earth. The most common approach to achieving contoured beams for geostationary satellites is the use of reflector antenna systems with shaped surfaces.

Conceptionally, the surface shaping is done by solving an optimization problem involving a function $F(\overline{X})$ where $F(\overline{X})$ computes the performance of the antenna system for the specific surface described by the variables $\overline{X}$, which define the surface of the reflector(s). Many realistic systems are electrically quite large, and need to be optimized over a range of frequencies, rendering analysis by full-wave methods outside the scope of modern computing. Further, solving the optimization problem is extremely difficult without access to analytical derivatives, which in general cannot be achieved with full-wave methods. Instead, asymptotic methods can be used, and by applying Physical Optics (PO), optionally supplemented by the Physical Theory of Diffraction (PTD), one can find the derivative matrix (also called the Jacobian matrix) of F analytically and efficiently.

Mathematically, the optimization problem to be solved can be described as

$$\min_{\overline{X}}. \quad F(\overline{X}) = \max\left(F_1(\overline{X}), F_2(\overline{X}), \ldots, F_m(\overline{X})\right), \tag{1a}$$

$$\text{s.t.} \quad \overline{\overline{A}}\,\overline{X} \le \overline{B}. \tag{1b}$$

Here, F_i denote the difference between the desired level of the quantity (gain, XPD, e.t.c.) being optimized and the actual value of the quantity at the i'th point in the coverage region. Thus, if e.g. $F_{62} = 0.1$, this means that the quantity at the 62^{nd} position is 0.1 dB worse than the desired level. Further, one can choose to specify constraints using the $\overline{\overline{A}}$ matrix, e.g. to constrain the surface from becoming too difficult to manufacture.

II. Improvements

The optimisation problem (1) can be very challenging, in part because of the non-linearity of F and in part because of the sheer size of the problem — it is not uncommon for the number of variables N to be several thousand, and the number of stations and constraints can be several hundreds of thousands. Further, each function evaluation can take several minutes, and the accuracy requirements are of order 10^{-6}.

A. *Optimisation*

Because of the costly evaluation of the objective function, choosing the correct methodology for the optimisation is crucial. In a recent development effort, we implemented a new solver based on the general framework described by Hald in [1], but applying modern convex optimization for the interior solver in a trust-region framework. The result is an algorithm with much higher speed and less memory use, particularly for cases with a large number of stations, variables and/or constraints. Further, when constraints are involved, a completely new solver has been written to determine a feasible starting point (or to rule out feasibility), vastly improving the feasible point algorithm used in Hald's work.

The algorithm has been discussed previously in the literature for single reflector systems [2] and reflectarrays [3]. However, its use for subreflector optimization is vastly different — in contrast to most single reflector systems, the time to evaluate the objective function is very high, and in contrast to reflectarrays, the number of unknowns is fairly modest. Thus, while the benefits of the new algorithm offer a nice improvement, particularly for cases with a large number of constraints, we wanted to further accelerate the computations.

B. *Subreflector Derivatives*

With the optimisation routine vastly improved, we turned our attention to accelerating the evaluation of the objective function F. One of the most time-requiring processes was the computation of derivatives for dual-reflector systems, because previously only the derivatives for the main reflector were available analytically. This meant that the derivatives of the subreflector surface had to be computed numerically by using the forward difference approximation — with a large number of variables, this process was very time-consuming.

To avoid this, we derived the exact analytical derivatives for both PO and PTD near-field and far-field expressions for the double-bounce interactions [4] occuring in dual-reflector

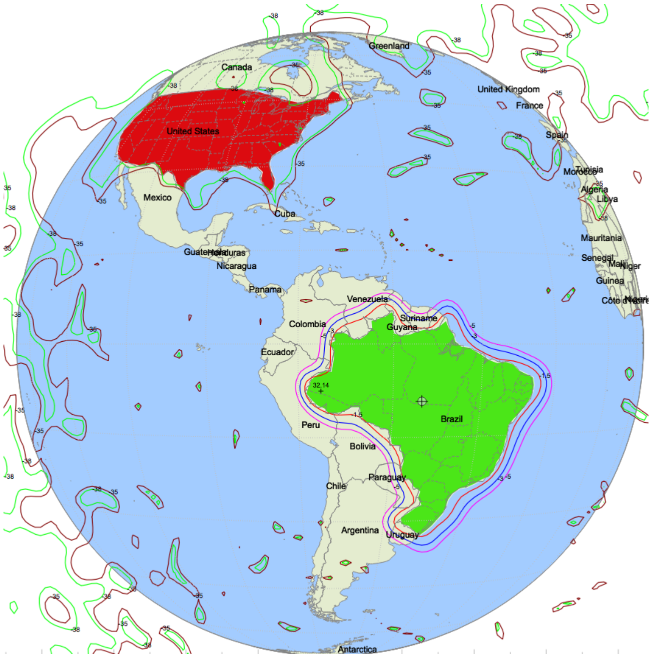

Fig. 1. The main coverage (in green) with high gain and low XPD goals over Brazil as well as (in red) the gain-suppresion region over the continental US. The field resulting from the optimization is shown superimposed with the $-1.5, -3, -5$ dB curves shown the contoured beam on Brazil as well as the -35 and -38 dB curves contoured around the US due to the gain-suppresion.

TABLE I
PARAMETERS FOR THE SYSTEM.

Frequency	12.49 GHz
Sub reflector, minor x major diameter	0.29 m x 0.3 m
Main reflector, minor x major diameter	2 m x 2 m
Feed taper at sub rim	-18 dB

systems. By computing these derivatives analytically, the cost of finding the Jacobian matrix is reduced significantly, particularly as the number of unknowns increase.

III. RESULTS

All results in this section have been produced on a Macbook Pro 2013 with 16 GB RAM.

To demonstrate the performance improvements, we consider a dual reflector system onboard a geostationary satellite located at $-70°$ longitude, and optimize for a Brazil coverage with goals set to provide high gain and low XPD in the coverage while suppresing the gain over the continental US. The coverage is shown in Figure 1, and $m = 20668$ stations are required to sufficiently characterize the coverage. The fundamental parameters of the antenna system are shown in Table I.

After some initial optimization runs to get a good starting point, we have a total of $N = 1442$ surface variables, of which slightly less than half are on the subreflector. We then run 200 iterations of the new and old minmax algorithms, both with the new analytical subreflector derivatives and the old numerical derivatives. The results are summarized in Table II, under the heading "No Constraints". Clearly, the use of the new algorithm presented in this paper (analytical derivatives, new optimization algorithm) leads to a significant speed-up relative to version 6.1 of POS [5] (numerical derivatives, Hald optimization algorithm) — a factor of 4.5 (297 min/65 min). The optimized value is the same for all four optimization runs.

TABLE II
COMPUTING TIMES IN MINUTES OF THE CASES IN SECTION III.

Algorithm	Numerical	Analytical
No Constraints		
Hald [1]	297	91
New	262	65
Constraints		
Hald [1]	364	173
New	266	71

We note that in this case, the biggest performance boost comes from the use of analytical derivatives on the subreflector. The improved optimization algorithm leads to about a 30% reduction in computing time on its own, but the use of analytical derivatives leads to a factor of 3-4 reduction in time compared to numerical derivatives.

A. Constraints

For demonstration purposes, we also add constraints to limit the local surface curvature to ensure that one can actually manufacture the optimized surface. A total of 61344 constraints are required. We run 200 iterations from the same starting point as when we did not have constraints — the results are summarized in Table II under the heading "Constraints". Again, we see a significant improvement in the new algorithm (71 min) versus POS 6.1 [5] (364 min), a factor of 5.1. The value is the same for all four optimization runs. As the table shows, the new derivatives lead to about a factor of 2-3 reduction in computing time, while the new optimization algorithm leads to a factor of 2 reduction as well.

IV. CONCLUSION

Surface optimization of dual reflector systems for contoured beams is a computationally challenging, but ubiquitous, part of the design cycle for many modern telecommunications satellite. By improving the optimization algorithm, as well as by implementing analytical expressions for the derivatives on the subreflector, we have been able to advance the state-of-the-art, reducing the optimization time by a factor of 5.

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