Efficient Monostatic RCS Calculation with Higher-Order MLFMM

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***Abstract*—The traditional $\mathcal{O}(f^6)$ computational time scaling when using Method of Moments for monostatic Radar Cross Section calculations is circumvented by applying the Multi-Level Fast Multipole Method (MLFMM) along with a number of modifications. The performance of the modified MLFMM-based algorithm is illustrated through a numerical example.**

I. Introduction

The computation of the monostatic Radar Cross Section (RCS) is an important task in many engineering applications. The computation time for full-wave methods, such as Method of Moments (MoM), scales as $\mathcal{O}(f^6)$, f being the frequency, and such methods have therefore traditionally been considered too computationally demanding. For the current state-of-the-art RCS prediction tools, the poor frequency scaling has resulted in algorithms [1], [2] that either relax the accuracy requirements by using asymptotic methods or, for implementations using full-wave methods, require extreme runtimes even on very advanced and expensive computing platforms [3].

The computational scaling reduces to $O(C(f,P)f^2 \log f)$ by applying the Multi-Level Fast Multipole Method (MLFMM), where $C(f,P)$ is the number of iterations required for convergence of an iterative solver, and P is the number of incidence angles. Despite this significant reduction in computing resources, most state-of-the-art full-wave RCS solvers avoid the use of MLFMM and instead prefer MoM, primarily since the number of iterations $C(f,P)$ can be very large due to a high number of incidence angles.

Here we present a range of developments towards an efficient algorithm for large-scale full-wave monostatic RCS, particularly for scatterers that are too large to handle with direct MoM. The algorithm includes a discretization based on higher-order basis functions and curved quadrilaterals, an MLFMM implementation aimed at keeping the memory requirements low, and a number of techniques reducing the total number of matrix-vector products needed for computing the RCS for many incidence angles.

II. Monostatic Radar Cross Section

The monostatic $\hat{\boldsymbol{\psi}}$-polarized RCS, $\sigma_{\psi\nu}(\theta_i,\phi_i)$, for a $\hat{\boldsymbol{\nu}}$-polarized incident field of a structure in the direction (θ_i,ϕ_i), is defined as

$$\sigma_{\psi\nu}(\theta_i,\phi_i) = \lim_{r\to\infty} 4\pi r^2 \frac{|\boldsymbol{E}^S(\theta_i,\phi_i)\cdot\hat{\boldsymbol{\psi}}|^2}{|\boldsymbol{E}^I_{\boldsymbol{\nu}}(\theta_i,\phi_i)|^2}, \tag{1}$$

where $\boldsymbol{E}^I_{\boldsymbol{\nu}}(\theta_i,\phi_i)$ denotes the electric field due to a $\boldsymbol{\nu}$-polarized plane-wave and propagation vector $\hat{\boldsymbol{k}} = -(\sin\theta_i\cos\phi_i\hat{\boldsymbol{x}} + \sin\theta_i\sin\phi_i\hat{\boldsymbol{y}} + \cos\theta_i\hat{\boldsymbol{z}})$, k is the free-space wavenumber, and $\boldsymbol{E}^S(\theta_i,\phi_i)$ is the scattered far field.

III. Solving the Integral Equation

For the considered perfectly electrically conducting scatterers, the mixed potential Electric Field Integral Equation (EFIE) – or the Combined Field Integral Equation (CFIE) for closed structures – is discretized using a higher-order approach. The surface geometry as well as the unknown surface current density are expanded using higher-order polynomials [4]. With this higher-order discretization, rather than one based on lower-order functions such as RWGs [5], the number of unknowns N required for obtaining a specific accuracy is significantly reduced.

A. *Multi-Level Fast Multipole Method*

To avoid the $N^3(f^6)$ term in the asymptotic scaling of MoM, the MLFMM can be used to perform the involved matrix-vector multiplications in $\mathcal{O}(N\log N)$ time and memory. Combining this with an iterative solver such as GMRES allows us to solve the MoM matrix equation in $\mathcal{O}(C(f,P)N\log N)$ operations. While the standard MLFMM for RWG basis functions is well studied, it is not straightforward to adapt MLFMM to a higher-order discretization. However, within the last couple of years, an efficient HO-MLFMM formulation has been developed [6], demonstrating significantly better performance than standard MLFMM and being very suitable for standard computer hardware.

IV. Using MLFMM for monostatic RCS

The MLFMM clearly reduces the memory footprint, but it is not clear that there is a reduction in runtime, since we have not yet quantified the number of iterations $C(f,P)$. Indeed, comparing our implementation to MoM-based RCS solvers such as [3], we clearly see that while MoM implementations focus on minimizing the number of unknowns N, our MLFMM implementation should focus on minimizing the number of iterations $C(f,P)$.

To quantify the number of iterations $C(f,P)$ if the angular range is ϕ_{int}, we first consider the dependence of P on f by considering the angular sampling density [7]

$$P = \frac{\phi_{\text{int}}}{\Delta\phi} = \frac{4f\rho_{\max}\phi_{\text{int}}}{c_0} \tag{2}$$

where c_0 is the speed of light and $\rho_{\max}$ is the maximum object radius in the observation plane. We note that P is the number of right-hand sides for each of the two polarizations of the incident plane wave, thus the total number of right-hand sides will be $2P$. Thus, we can express $C(f, P)$ as

$$C(f, P) = 2N_{\text{it}}P, \tag{3}$$

where N_{it} is the number of iterations required for the iterative solver to converge for each of the $2P$ right-hand sides.

A. Interpolating the monostatic RCS

Usually, a larger number of incidence angles P_u are needed than required by the sampling criterion (2), and the monostatic RCS can then be interpolated by one of the many well-studied methods for interpolation of functions on a sphere. When $P_u > P$ it is computationally much more efficient to solve the MoM matrix equation with P right-hand sides and subsequently interpolate rather than solving it directly with P_u right-hand sides.

B. Iterative Solver

The most popular iterative solver for MLFMM problems appears to be the GMRES [8], which in its basic formulation is a Krylov method based on a single right-hand side. Hence, a standard textbook RCS implementation does not utilize the fact that the P right-hand sides are related. To improve on this, we have implemented a Block-GMRES solver as discussed in [9], which minimizes all P residuals simultaneously. A Block-Krylov solver provides a lower number of matrix-vector products than the $N_{\text{it}}P$ estimate given in (3) by utilizing information from all the P right-hand sides. In addition, our implementation employs deflation, which reduces the dimension of the Krylov subspace from P columns to P_d columns by applying a rank-revealing decomposition to the orthogonalized residual space.

V. Numerical Result

The monostatic RCS is calculated for an F16 fighter at 3 GHz using CFIE. $P_u = 1800$ incidence angles are desired and the number of right-hand sides after deflation is $P_d = 763$ per polarization. The applied quadrilateral mesh is seen in Fig. 1. The simulation requires 301681 unknowns, 14 GB storage, and 11 iterations and 10.2 hours to converge on a Dual Intel Xeon E5-2690 2.6 GHz computer with a total of 24 cores. No symmetry assumptions are used in the simulation and the result is shown in Fig. 2. For comparison the RCS of the same fighter at the same frequency is performed in [3] using a direct MoM solution on an Intel Xeon E5-2660 2.2 GHz with 20 cores, 3 GPU cards, and 4 SSD disks. The simulation time is 9.8 hours and symmetry is used to half the number of unknowns to 160411. With the use of symmetry the storage requirement of the MoM-based solution is at least 96 GB. For non-symmetric configurations the required storage is at least 384 GB.

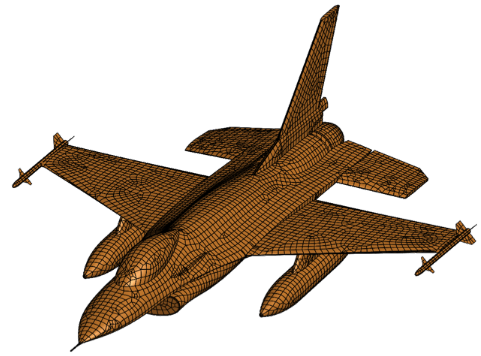

Fig. 1. The higher-order quadrilateral mesh of the F16 fighter.

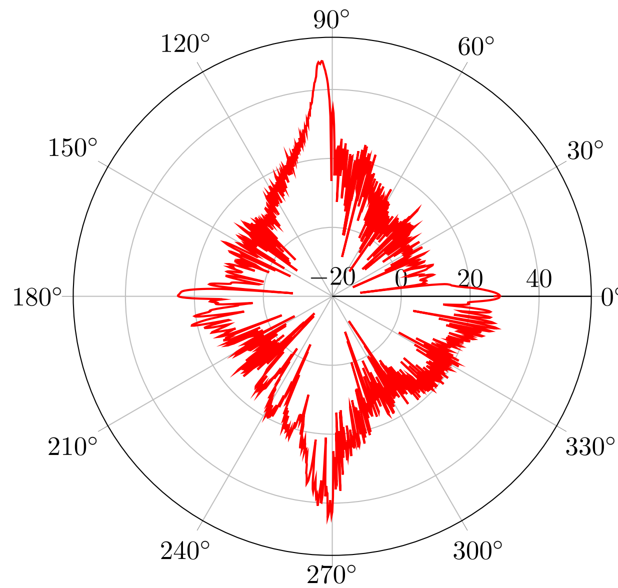

Fig. 2. F16 monostatic RCS in dB at 3 GHz in the observation plane in which the plane of the tail is contained. 0° is the front and 180° the rear of the fighter.

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