New Analysis Capabilities for Electrically Large Antennas and Platforms

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***Abstract*—We present recent advances in the RF modeling of electrically large antennas and platforms, with an emphasis on reflector antennas and satellite platforms. The Higher-Order Multilevel Fast Multipole Method (HO MLFMM) was introduced recently in the commercial software tool GRASP, and this algorithm is employed in this paper due to the very low memory requirements and the need for very high accuracy. We show that the memory distribution of HO MLFMM is different from the typical low-order MLFMM, thereby enabling a more efficient out-of-core solution which allows very large problems to be solved with modest memory requirements. We also investigate the challenging problem of MLFMM-based computation of waveguide scattering parameters when several horn feeds are mounted on large satellite platforms. We show that the use of a Block GMRES method provides a faster and/or more accurate solution than the standard GMRES solution method.**

***Index Terms*—Multi-Level Fast Multipole Method, higher-order Method of Moments, full-wave methods, commercial software.**

I. Introduction

A fast and accurate full-wave method for electrically large problems is needed in several applications, including satellite communication systems or scientific missions. Space applications typically require very high accuracy and a desired dynamic range exceeding 100 dB is not uncommon. At the same time, the technological advances imply that multiple antenna systems can be tightly packed on the satellite platform, thus requiring full-wave based computation of the EM interaction between several antenna systems and the platform. The demands from the industry require that larger systems must be analysed in a shorter time, and with higher accuracy than previously possible. Furthermore, there is a need for computation of the entire scattering matrix when multiple antenna systems are colocated on the same platform.

In this paper, we show that the Higher-Order MLFMM introduced recently [1] is capable of reaching a very high level of accuracy with a very small memory footprint. Therefore, the HO MLFMM is more suitable for analysis of electrically large platforms than a standard MLFMM implementation [2]. At the same time, the high current expansion order implies that a relatively large part of the memory is occupied by the near-interaction matrix whereas the memory required for the MLFMM acceleration (basis function patterns, group patterns, translation operators, etc) occupies a smaller part of the memory. This property is important when an out-of-core MLFMM solution is applied to extend the range of solvable problems on a given computer hardware. We investigate the memory distribution of the HO MLFMM and illustrate the performance of the out-of-core HO MLFMM which was recently made available in the GRASP software.

Computation of scattering matrices, e.g. for accurate assessment of coupling between multiple feed horns on an electrically large platform, can be efficiently performed with HO MLFMM. However, the MLFMM problem must be solved for N right hand sides (RHS) to compute an $N \times N$ scattering matrix, which implies that the iteration time can be quite long. Several approaches for MLFMM solutions with multiple RHS have been developed for monostatic RCS applications but these methods are not applicable when there is no correlation between the various RHS. In order to reduce the iteration time for scattering matrix computations, we have introduced the block GMRES Krylov solver into the HO MLFMM algorithm. We show that this solver reduces the iteration time and provides higher accuracy at the same time.

II. Memory and CPU Requirements of HO MLFMM

The HO MLFMM algorithm presented in [1] is able to work with basis functions of very high order, e.g. 5th order, with very modest memory requirements. Unlike this new algorithm, the commonly used standard MLFMM algorithm will result in a very high memory consumption when high expansion orders are used, due to the larger geometrical patch sizes employed. These properties are illustrated in Figure 1, showing the RMS error obtained when computing the bistatic radar cross section of a conducting sphere, using the Combined Field Integral Equation (CFIE). The results are reported for varying polynomial orders p and for both standard MLFMM and HO MLFMM. For standard MLFMM, it can be seen that the $p = 2$ curve is closest to the lower left corner of the plot and hence, second order basis functions provide the best trade-off between accuracy and memory. In contrast to this, the blue curves are almost identical for $p > 1$, indicating that the HO MLFMM reduces the memory requirements for all expansion orders, but in particular for high expansion orders that can be applied without a memory penalty. Figure 1 also shows that HO MLFMM is far more suitable than standard MLFMM when high solution accuracy is needed, since the error can be reduced by one order of magnitude with only a small increase in memory.

The HO MLFMM memory consumption reported in Figure

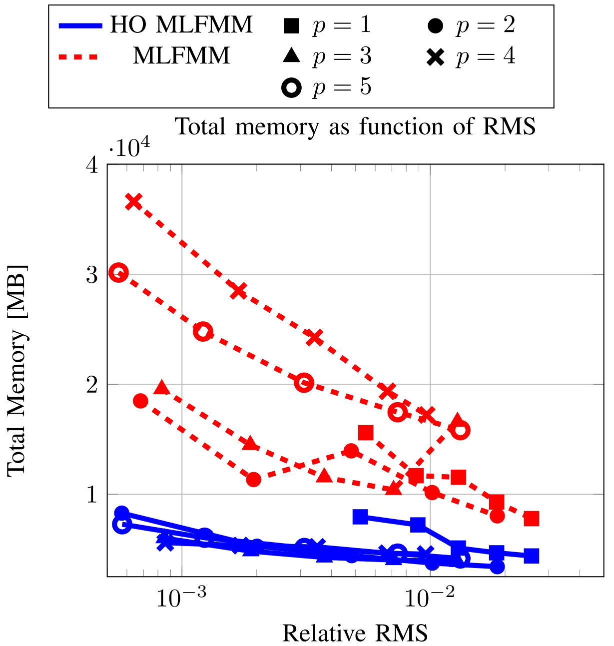

Fig. 1. The total memory for varying RMS error and polynomial order p, using CFIE, with the standard MLFMM algorithm and the modified HO MLFMM algorithm.

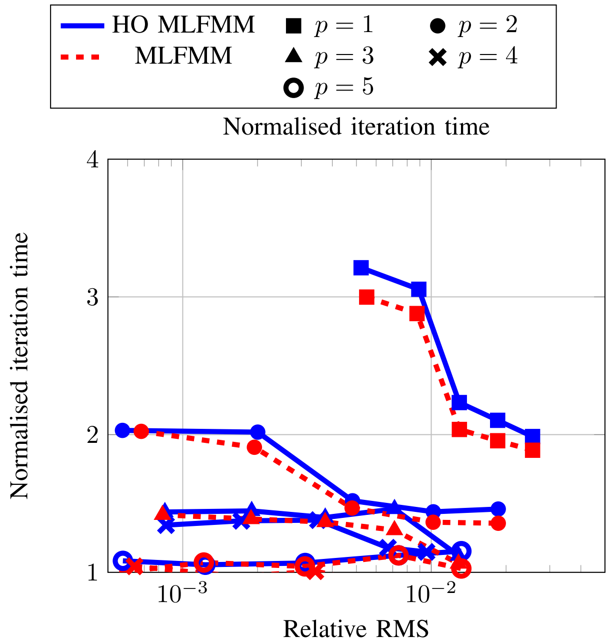

Fig. 2. Time per matrix-vector product normalised to the time for the fastest run. Each curve corresponds to a fixed polynomial order (p= 1, 2, 3, 4, 5) and results are shown for the standard algorithm (red) and the modified algorithm (blue).

1 shows that all expansion orders higher than $p = 1$ leads to roughly the same memory-versus-accuracy tradeoff. However, there is still a strong motivation for using the highest possible expansion order which can be understood by studying the CPU time required for the iterative solution reported in Figure 2. The CPU time is normalised to the time required for the fastest run and we report the total iteration time, implying that the curve shows the combined effect of a varying time per matrix-vector product as well as a different number of iterations. It can be observed that there is a direct relation between the expansion order and the time required for the iterative solution: A high expansion order leads to the shortest possible iteration time. In addition, we note that a high expansion order is far more suitable than a low expansion order when a high solution accuracy is needed, because the curve corresponding to the highest order is almost flat.

III. Out-of-Core HO MLFMM Solver

The favorable memory and CPU consumptions of the HO MLFMM algorithm imply that electrically larger problems than previously possible can be solved. Nevertheless, there is a strong demand for solving even larger problems, and often problems requiring more memory than available. This need can be addressed by developing an Out-of-Core (OoC) solver that employs disk storage instead of RAM, thereby reducing the peak memory requirement at the expense of a longer solution time. Direct OoC solvers have been developed for non-accelerated higher-order MoM solvers [3] but these cannot handle electrically large problems due to the poor scaling with problem size. Low-order MLFMM solvers with OoC capabilities have also been developed, e.g. [4], but as demonstrated in the previous section, low-order MLFMM requires significantly more memory and longer iteration time than HO MLFMM. In addition, we note the following difference between the low-order and the higher-order MLFMM:

- For low-order MLFMM, approximately 50% of the memory is used for the near matrix and the basis functions patterns. The remaining part of the memory os occupied by group patterns, translation operators, and other MLFMM data that is needed several times in each iteration.
- For HO MLFMM, approximately 75% of the memory is used for the near matrix and the basis functions patterns. Since a larger part of the memory is occupied by data that is only needed once per iteration, the HO MLFMM is more suitable for an OoC solution than low-order MLFMM.

The difference outlined above and the expansion order have a direct impact on the amount of data that can be stored on disk and the data that must be kept in RAM. This is illustrated in Figure 3 that shows the amount of RAM needed when the OoC HO MLFMM is used to solve the same test case as studied in the previous section. Hence, the plot is similar to Figure 1, but here we report only the memory that cannot be swapped to disk when the OoC algorithm described in [5] is used. Two observations can be made:

1) The memory consumption decreases significantly as the expansion order increases.
2) The curve for the highest expansion order $p = 5$ is almost flat, indicating that the memory requirement is independent of the solution accuracy. This strong

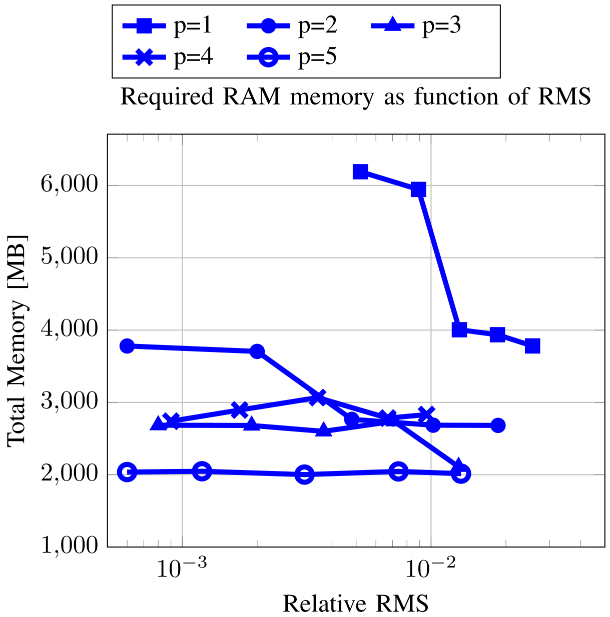

Fig. 3. Memory (RAM) required by the out-of-core HO MLFMM solver for various expansion orders.

TABLE I
OUT-OF-CORE PERFORMANCE ON A PARABOLOIDAL REFLECTOR WITH DIAMETER D.

D [λ]	In-core		Out-of-Core	
	Memory [GB]	Time	Memory [GB]	Time
50	0.68	1:11 min	0.11	1:37 min
100	2.60	5:05 min	0.32	6:43 min
200	10.25	24:11 min	1.16	40:00 min
400	20.08	**N/A**	4.42	4:11 hrs

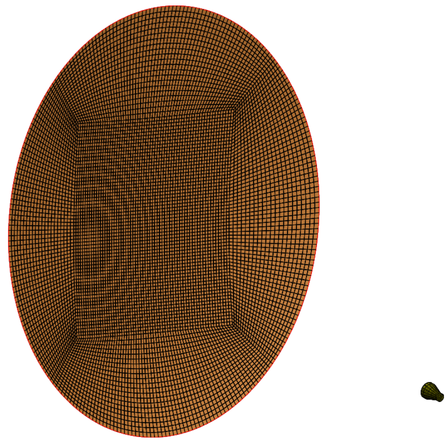

Fig. 4. Mesh of the offset $D = 100\lambda$ paraboloidal reflector, illuminated by an axially corrugated horn, used as a test case for the Block GMRES solver. Note that each patch has roughly 1.5λ sidelength.

property implies that high accuracy can be obtained on modest computers.

The last observation above can be understood by considering the effect of improving the accuracy by p-refinement instead of h-refinement. When the polynomial order p is increased, additional basis functions will be defined on each patch whereas the size of the patches and the MLFMM grouping are unchanged. The increased number of basis functions has a direct impact on the lowest level of the MLFMM tree, i.e., the size of the near-matrix, the preconditioner memory, and the basis functions patterns. However, all these quantities are placed in OoC storage. The data associated with the higher MLFMM levels are kept in RAM but these are unchanged when doing p-refinement. Therefore, the OoC HO MLFMM allows very high solution accuracies with very little impact on memory requirements.

The performance of the OoC algorithm is now illustrated by considering a paraboloidal reflector with different diameters between 50λ and 400λ. The radiation patterns have been computed with HO MLFMM on a laptop with 16 GB RAM. The memory and CPU time are reported in Table I for both the in-core and the out-of-core solution. It can be observed that the required RAM is between 5 and 10 times lower with the OoC solver, at the expense of a longer runtime. This example illustrates that the OoC HO MLFMM is very suitable for solving electrically large problems with high accuracy, using only a modest computing platform.

IV. Block GMRES for HO MLFMM with Multiple Right-Hand Sides

When computing the currents excited on a structure by multiple distinct excitations, e.g., when the scattering matrix of a multiport antenna system is required, a system of equations with multiple right-hand sides is set up

$$\overline{\overline{Z}}\,\overline{\overline{I}} = \overline{\overline{V}} \tag{1}$$

where $\overline{\overline{V}}$ and $\overline{\overline{I}}$ have N rows and P columns. When N is large, these problems are typically solved by P consecutive applications of an iterative linear solver. In some applications, the convergence for the $i+1$'st right-hand side can be improved by using information from the solution of the i'th right-hand side. Unfortunately, these tricks cannot be applied when the excitations are uncorrelated. A more thorough approach is the use of Block Krylov solvers - these solve all P systems simultaneously, requiring P matrix-vector products with $\overline{\overline{Z}}$ per iteration, but compressing all the information from those P systems into one Krylov subspace. One such method, the Block GMRES, is described in detail in [6] and this method has been implemented in the HO MLFMM solver.

As an example of the properties of the Block GMRES, we consider an offset paraboloidal reflector with circular projected rim, illuminated by an axially corrugated horn designed for use in the 20-30 GHz range. We fix the frequency at 30 GHz, which yields an overmoded horn, and therefore we include 20 waveguide modes in the scattering matrix computation, to ensure that the higher-order modes are not detrimental to the performance. The scenario is illustrated in Figs. 4-5, and the total number of right-hand sides in the system is $P = 20$.

The performance of the Block GMRES solver is contrasted

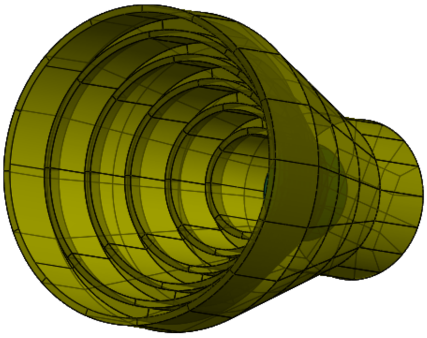

Fig. 5. Close-up of the axially corrugated horn from Fig. 4.

TABLE II
NUMBER OF MATRIX-VECTOR MULTIPLIES

Method	Number of runs	Number of iterations	Matrix-vector products per iteration	Total
GMRES	20	13-22	1	354
Block GMRES	1	14	20	280

with the standard P applications of non-restarted GMRES in Table II, with the relative residual tolerance of 10^{-3}. The key performance parameter is the number of matrix-vector products required for all P right-hand sides to achieve convergence - for the standard GMRES, this is 354, while Block GMRES is 280. Thus, the Block GMRES reduces the total number of matrix-vector products with about 20% for this specific case.

An additional benefit, however, is that the use of Block GMRES often leads to more accurate solutions for many of the P systems. The Block GMRES solver is stopped when all P systems have achieved the requested relative residual tolerance - at that point, some of the systems might have converged several iterations ago. Figure 6 shows the achieved relative residual error, demonstrating that not only does the Block GMRES require fewer iterations for all systems to achieve convergence, the final achieved result will also be much more accurate - some of the systems have converged even at the 10^{-4} threshold. This implies that the scattering matrix computation is both faster and more accurate.

V. CONCLUSION

We have shown that the HO MLFMM algorithm is very suitable for electrically large problems due to a very low memory and CPU requirement. The difference between the commonly applied low-order MLFMM and HO MLFMM is particularly pronounced when high accuracy is desired, because a higher expansion order provides a significant reduction of the discretisation error while resulting only in a minimal increase of the required memory. We have also shown that the HO MLFMM results in a relatively large near-matrix which makes the algorithm suitable for an out-of-core solution. When the out-of-core HO MLFMM is used, a very high solution accuracy can be obtained by p-refinement without affecting the

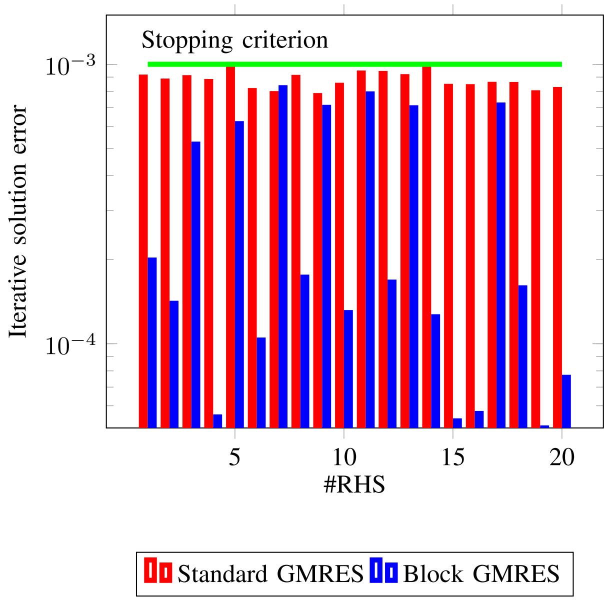

Fig. 6. Achieved relative residual convergence for the $P = 20$ right-hand sides.

required in-core memory. Finally, we have shown that a Block GMRES solver can be used to reduce the iteration time when the HO MLFMM is applied for computation of scattering matrices, e.g., when multiple horn antennas are embarked on an electrically large satellite platform. The algorithmic improvements of the HO MLFMM discussed in this paper have recently been introduced in the commercial software GRASP.

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