Beam-Waveguide Analysis Using Complex Conical Beams

S. Skokic+, M. Casaletti#, S. Maci# and S. Sorensen*

+*Faculty of Electrical Engineering and Computing, Unska 3,*
HR-10000 Zagreb, Croatia
sinisa.skokic@fer.hr

Department of Information Engineering University of Siena,
Via Roma, 56 - 53100- Siena, Italy
{casaletti, macis}@unisi.it

**TICRA*
Læderstræde 34, DK-1201 Copenhagen K, Denmark
sbs@ticra.com

***Abstract*— This paper presents a method of analysing beam waveguide systems, based on Physical Optics and Complex Conical Beams. The analysis procedure for one reflector consists of calculating the field spectrum in the auxiliary input/output plane, expanding the field (spectrum) into complex conical beams, propagating the beams to the subsequent reflector and calculating the reflected fields in the next auxiliary plane via Physical Optics. The method is demonstrated on a two-reflector system. Optimal position of the auxiliary plane and efficient calculation of the field spectrum are discussed. Huygens' source is employed as excitation, in order to generate physically correct incident fields with a Gaussian taper. The results show good agreement with the reference (pure Physical Optics) solution.**

I. Introduction

The main characteristic of the quasi-optical frequency range, spanning from the upper end of the microwave range and all the way up to terahertz frequencies, is that the devices interacting with the electromagnetic wave are much larger than a wavelength, but still not large enough to be able to take full advantage of the simplifications that optical systems and devices benefit from. The term "simplifications" here refers both to the analysis of such systems and the electric and magnetic properties of matter. For example, at optical frequencies there is almost no interaction between the electromagnetic wave and the molecules of the optical fibre core, resulting in very low losses of such structures. Similarly, at microwave frequencies, electro-magnetic waves can be conducted with minimal losses using waveguides, owing to the very low losses in waveguide walls.

In the quasi-optical region, on the other hand, neither of the two conditions is satisfied, and the losses in both metallic and dielectric guiding structures can be as high as 0.5 dB/cm [1]. Therefore, the idea is to conduct the electromagnetic wave from the generator to the main radiating element through air. To ensure minimal leakage of energy (i.e. minimal losses), the wave is directed in space and kept focused by a sequence of curved, shaped metallic reflectors, called a beam waveguide, as shown in Fig. 1. However, such a solution for conducting the electro-magnetic wave presents significant challenges both from practical and analytical aspect.

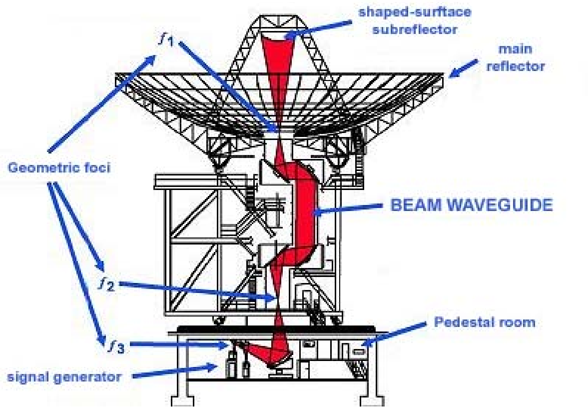

Fig.1: Beam waveguide system for conducting the electromagnetic wave from the generator to the antenna.

The analysis of reflector antenna systems at millimetre-wave frequencies requires the development of new methods particularly suited for that frequency and size range. The analysis methods usually seek to express the radiated field in terms of a relatively low number of wave objects of higher complexity and in this way reduce the number of unknowns in the system. The aim is to develop a modular process, whereby the total reflected field from one reflector can be re-expanded into a new sum of wave objects of the same kind, and the whole procedure can be repeated in the same way for all subsequent reflectors. Recently, we introduced a new beam formulation [2]-[3] and introduced the basic steps for analysing reflection by a single antenna element [4]. In this paper, we round up the analysis and demonstrate the application of the developed method to a system of two large ellipsoidal reflectors illuminated by a Huygens' source. The formulation and some preliminary results are shown in the following sections.

II. Formulation

The geometry of the analysed two-reflector system is shown in Fig. 2. For simplicity, both analysed antennas are ellipsoidal reflectors. Another reason for working with ellipsoidal reflectors is that if the source is placed in the first focal point of a reflector, upon reflection the reflected beam will focus in the plane containing the second focal point, which is optimal in the sense of keeping the beam tight. The frequency is 3 GHz, i.e. $\lambda = 0.1$ m, and the semi-axes of both ellipsoids are $a = 5$ m $= 50\lambda$, $b = c = 4$ m $= 40\lambda$. The first reflector is illuminated from its geometrical focus F1 by a complex Huygens source with a beam taper of $A = 10$ dB and a beam taper angle of $\theta = 6°$, which corresponds to the beam waist $w_{0,inc} = 0.326$ m $= 3.26\lambda$. The incident beam is launched towards the apex of the first ellipsoid (point (0, 0, 4 m) on the z-axis).

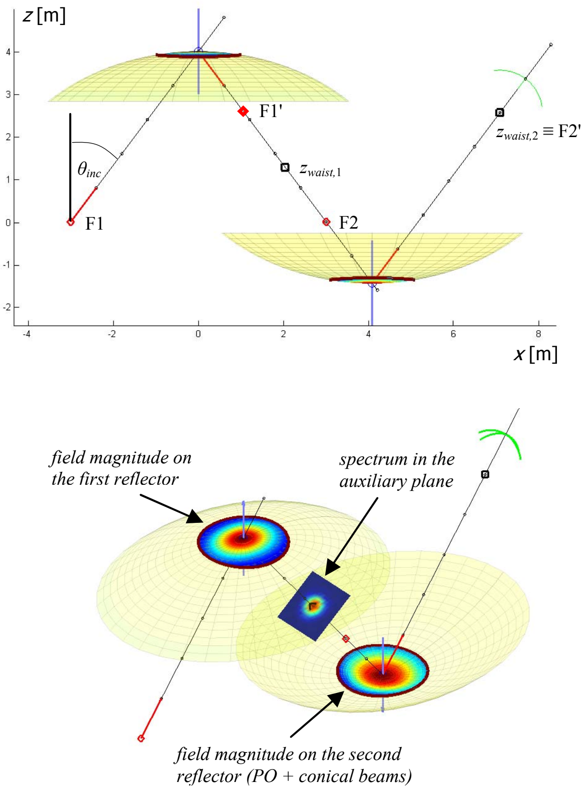

Fig.2: Side view (top) and perspective view (bottom) of a system of two ellipsoidal reflectors with a circular rim. Green lines denote the far-field observation points.

The analysis of a beam waveguide using wave-objects consists of the following basic steps, which must be done for each reflector (the first reflector may not need step 1 below and the analysis of the last reflector obviously stops at point 3):

1) calculate field spectrum in the input plane
2) expand the incident field into complex conical beams
3) compute equivalent currents on the reflector surface using complex conical beams
4) calculate reflected field via Physical Optics

In the test cases, the first reflector was illuminated by a Huygens' source. The Huygens' source consists of an electrical x-directed short dipole (a Hertzian dipole) and a y-directed short magnetic dipole, and produces a z-directed beam (in local coordinate system). When this source is located at a point with the complex position $(x, y, z) = (0, 0, jb)$, the beam it generates will have a Gaussian taper, similar to that of real sources employed in quasi-optical reflector systems (e.g. corrugated horn antennas). The far-field of a Huygens' source with a complex displacement b is

$$\vec{E}(r,\theta,\phi) = E_0 \frac{e^{-jkr}}{r} e^{kb\cos\theta}(1+\cos\theta)(\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\theta}). \tag{1}$$

The advantage of a Huygens' source over simple Gaussian beams lies in the fact that the field it generates is physically correct in both near field and far field, and is co-polar, in all directions, to the linear x-polarisation as defined according to Ludwig's 3rd definition [8]. If a beam with a taper of A decibels at an angle θ_t is desired, the necessary value of complex coordinate b is calculated from (1) as

$$b = \frac{20\log\Big((1+\cos\theta_t)/2\Big) - A}{20k(1-\cos\theta_t)\,\log(e)}. \tag{2}$$

The propagation of complex conical beams (CCB) has been described in detail in previous communications [2]-[4]. In order to analyse a realistic system of reflectors, however, there are additional problems that need solving. For instance, one has to determine the optimal position for the input/output plane for a given system geometry. By input/output plane we understand the plane in which the field is translated to CCB's. The goal is to do that where the beam is the most focused, so as to reduce as much as possible the size of the sampling surface in that plane. Another good consequence of such a choice of input/output plane is that the computation of the Physical Optics integral is the fastest there [5], since the reflected field is most in-phase in the plane where it is focused. Two possible choices for the input/output plane ("auxiliary plane" in Fig. 2) are investigated in Sec. 3.

Another important aspect of the system analysis is the efficient calculation of the electric field spectrum in the input/output plane. Typically, a 2D Fast Fourier Transform is employed for that purpose. However, it should be noted that the standard FFT algorithm samples the field on a rectangular grid, and moreover returns the calculated spectral points also on a rectangular grid. On the other hand, the CCB expansion requires spectrum samples on a polar grid, which in return means that either an additional 2D interpolation or a significant increase in the number of sampled points are necessary, the former translating into a somewhat lower accuracy of the algorithm, and the latter into an increase in calculation time. A better alternative to the classical FFT is the recently introduced Pseudo-Polar FFT algorithm (PPFFT) [9], which employs a series of 1-D FFT transformations tor calculate the spectral data on a polar grid which is uniform in radial direction and almost uniform in azimuthal direction.

The third possible way of calculating the spectrum for the CCB expansion is the use of inverse Near-Field to Far-Field transformation (NF-FF) [10], [11]. Here, the reflected field samples are calculated on a spherical surface in the estimated far-field of the outgoing beam and the spectrum in the input/output plane is then computed in closed-form. The two latter approaches (PPFFT and NF-FF) have been considered in the numerical verification.

Other important points of the algorithm include a criterion for truncating the FFT-GPOF expansion used in the CCB expansion and efficient handling of the complex geometry of multiple reflector systems. The latter implies the definition of three coordinate systems at every reflection (global, reflector-based and beam-based) and the development of fast and simple routines for converting field values and vectors between coordinate systems.

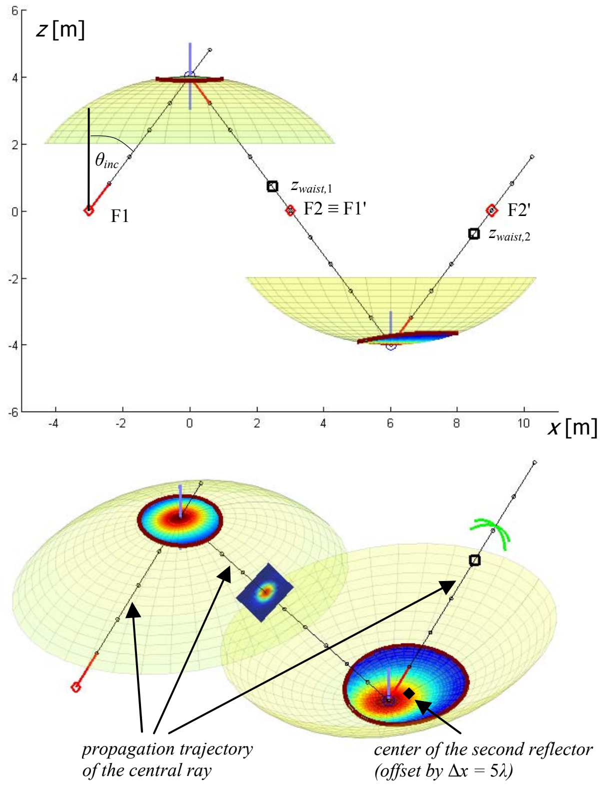

Fig.3: Side view (top) and perspective view (bottom) of a system of two ellipsoidal reflectors (second test case).

III. Numerical Results

For numerical tests, we considered two scenarios. The first test case, illustrated in Fig. 2, was created such that the two reflectors were symmetrical with respect to the actual focus of the reflected beam (point $z_{waist,1}$ in Fig.2). In the second test case, illustrated in Fig. 3, the second part of the two-reflector system was altered, and the *z*-axis apex of the second ellipsoid was set to the point (6 m, 0, 4 m). It can be noted that with such a placement of the second ellipsoid, the two ellipsoids are actually geometrically symmetric around the focal point F2 of the second reflector, which is considered optimal from the ray-analysis perspective (because F2 ≡ F1'). Therefore, the two selected test scenarios allow for a comparison of optimal reflector positioning from the beam analysis viewpoint (first case) and the ray-optical analysis viewpoint (second case).

Another modification introduced in the second test case is an increase in the second reflector surface $r_0' = 15\lambda$ and a small offset of the second reflector centre ($\Delta x = 5\lambda$; by default, the rim centre is set to coincide with the intersection of the central ray and the reflector surface, to achieve the most symmetric field distribution on its surface). Comparing Figs. 2 and 3 (bottom images), one can see that in Fig. 3 the fields on the inner part of the second reflector are truncated more abruptly than in Fig. 2. Consequently, for the second test case one can expect to have more ripples in the radiation pattern in the $\phi = 0°$ cut-plane.

The results are compared to the reference PO solution obtained via GRASP SE and are presented in Figs. 4 thru 7. In all plots, the reference axis for angle θ is the estimated direction of propagation of the wave upon final reflection, obtained by ray analysis (shown in Fig. 3).

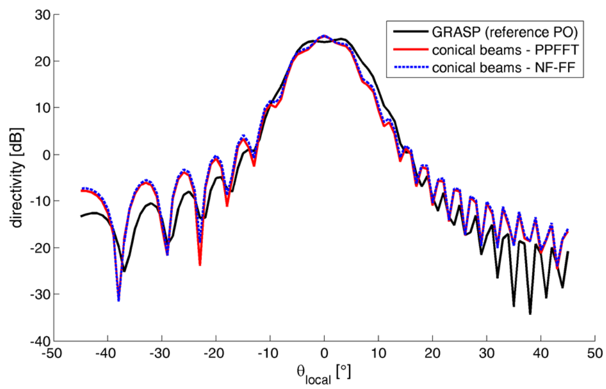

Fig.4: Radiation pattern (directivity) of the output beam after two reflections (first test case); $\phi_{local} = 0°$.

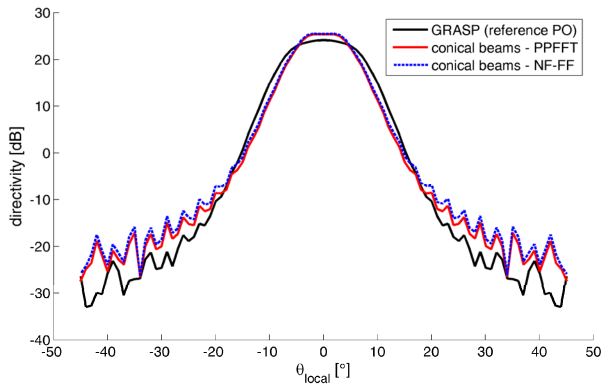

Fig.5: Radiation pattern (directivity) of the output beam after two reflections. (first test case); $\phi_{local} = 90°$.

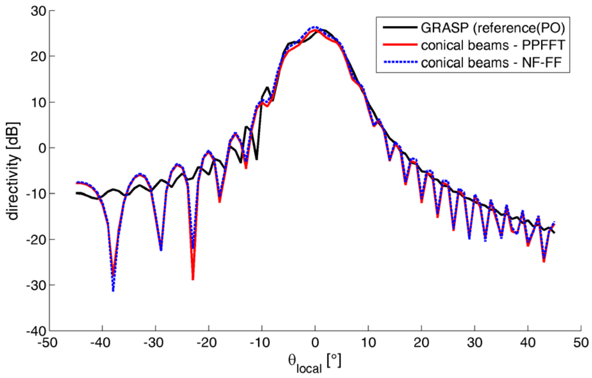

Fig.6: Radiation pattern (directivity) of the output beam after two reflections (second test case); $\phi_{local} = 0°$.

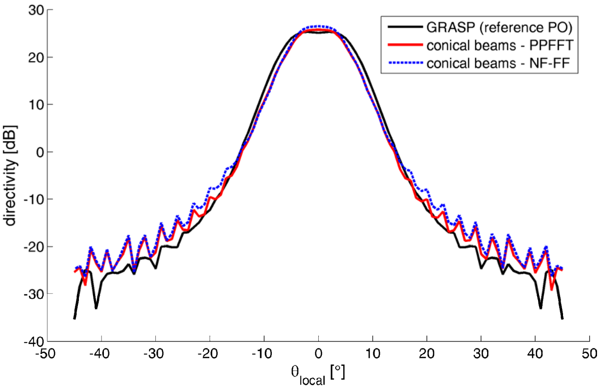

Fig.7: Radiation pattern (directivity) of the output beam after two reflections. (second test case); $\phi_{local} = 90°$.

The agreement is seen to be very good within a dynamic range of 40 dB, proving the applicability of this analysis method. As expected, for the second test case the field distribution has fewer ripples in the $\phi_{local} = 90°$ cut-plane (Fig. 7) than in the $\phi_{local} = 0°$ cut-plane (Fig. 6). The curve in Fig. 7 is also smoother than the corresponding field distribution of Fig. 5 (for the first test case), owing to the larger radius of the second reflector.

We also note that the two alternative approaches to spectrum calculation, described in Sec. 3, yield results of similar accuracy. Both approaches have been found to be significantly faster (approximately 10 times) than the pure PO-based analysis.

IV. CONCLUSION

A method of analysing beam waveguide systems based on a combination of Physical Optics and Complex Conical Beams has been presented and a two-reflector system has been analysed. A Huygens' source was employed as excitation, in order to generate physically correct incident fields with a Gaussian taper. Two methods of computing the spectrum in the plane where the field is expanded into complex conical beams have been compared, yielding results of similar accuracy. Overall, the results are comparable to the referent Physical Optics results, while the computation times have been reduced by a factor of ten, proving the applicability of the developed method.

REFERENCES

[1] P. F. Goldsmith, "Quasi-Optical Techniques," *Proc. IEEE*, vol. 80, pp. 1729-1747, Nov. 1992.

[2] S. Skokic, M. Casaletti, S.Maci and S. Sørensen "Complex Conical Beam Expansion for the Analysis of Beam Waveguides", Proc. 3rd European Conference on Antennas and Propagation (EUCAP 2009), Berlin, Germany, 2009.

[3] S. Skokic, M. Casaletti, S.Maci and S. Sørensen "Complex Conical Beams for Aperture Field representations", *IEEE Trans. Antennas & Propagation*, accepted for publication (2010).

[4] M. Casaletti, S. Skokic, S.Maci and S. Sørensen "Beam Expansion in Multi-reflector Quasi-Optical Systems", Proc. 4th European Conference on Antennas and Propagation (EUCAP 2010), Barcelona, Spain, 2009.

[5] T. Bondo and S. Sørensen " Physical Optics Analysis of Beam Waveguides Using Auxiliary Planes", *IEEE Trans. Antennas Propagat.*, vol. 53, No. 3, Mar. 2005, pp. 1062-1068

[6] N. J. McEwan and P. F. Goldsmith, "Gaussian Beam Techniques for Illuminating Reflector Antennas", *IEEE Trans. Antennas & Propagation*, vol. 37, No. 3, pp. 297-304, 1989

[7] H-T. Chou and P. H. Pathak, "Uniform Asymptotic Solution for Electromagnetic Reflection and Diffraction of an Arbitrary Gaussian Beam by a Smooth Surface with an Edge", *Radio Science*, vol. 32, no. 4, pp. 1319-1336, 1997.

[8] K. Pontoppidan (Ed.), *GRASP9 Technical Description*, TICRA Engineering Consultants, Copenhagen, Denmark, 2005.

[9] A. Averbuch, R. R. Coifman, D. L. Donoho, M. Elad and M. Israeli, "Fast and Accurate Polar Fourier Transform", *Appl. Comput. Harmon. Anal.*, vol. 21, p.p.145–167, 2006.

[10] C. A. Balanis, *Antenna Theory – Analysis and Design*, 2nd edition, John Wiley and Sons, NY, USA, 1997.

[11] J. J. H. Wang, "An Examination of the Theory and Practices of Planar Near-Field Measurement", *IEEE Trans. Antennas & Propagation*, vol. 36, no. 6, pp.746-753, 1988.