

# Analysis of Printed Reflectarrays Using Extended Local Periodicity

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**Abstract**—An analysis technique for improved modeling of a printed reflectarray is proposed. The technique is based on a periodic approach where periodicity is applied on an extended unit cell, which includes the actual elements surrounding the element under consideration. An offset reflectarray sample has been manufactured and measured, and comparison of simulations and measurements is presented to verify the proposed technique.

## I. INTRODUCTION

Printed reflectarrays are a promising candidate for realizing low-cost high-gain antennas and have been the subject of significant research interest in recent years [1], [2]. A reflectarray combines some of the best features of reflector and array antennas. It consists of a flat surface with many printed elements illuminated by a feed. The elements are designed to reradiate the incident field such that a specified far-field pattern is obtained. The reflectarray eliminates the need for a bulky, expensive and relatively high-loss feeding network required by conventional array antennas, and at the same time circumvents the requirement of curved surfaces in a conventional reflector antenna. Several types of advanced reflectarrays exist such as dual/multi-band reflectarrays, contoured/multi beam reflectarrays, and reconfigurable reflectarrays [2].

An accurate and efficient analysis technique is important to precisely predict the radiation properties of reflectarrays, and essential for optimization purposes. Currently, the state-of-the-art reflectarray modeling algorithms cannot reach the same level of accuracy as that of commercially available reflector and array analysis software packages, e.g. GRASP [3].

To ensure high-gain performance, the electrical size of reflectarrays is usually very large. Recently, efficient Method of Moments (MoM) simulation techniques using acceleration methods have been applied on entire reflectarrays to analyze their radiation features [4]. However, the most efficient analysis method is based on a periodic approach where each element in the reflectarray is analyzed by assuming local periodicity (LP), that is, the individual array element is embedded in an infinite array of identical elements. The spectral domain Method of Moments (SMoM) assuming local periodicity is efficient for reflectarrays made of varying-sized patches, and many advanced reflectarrays have been designed using this

technique [2]. However, reflectarrays are inherently aperiodic due to the need to compensate the spatial phase delay from the feed, and the local periodicity assumption gives rise to discrepancies when measured and simulated radiation patterns are compared. This paper proposes a solution that attempts to reduce these discrepancies.

## II. REFLECTARRAY ANALYSIS

### A. Spectral Domain Method of Moments

The SMoM used in this paper is based on a multilayer formulation as described in [5]. For a single layer case, the formulation corresponds to that used in e.g. [6].

It has been shown in [7] that the convergence of the SMoM solution for resonant printed reflectarray elements becomes poor and singular basis functions with correct edge conditions are required for accurate characterization of the reflectarray elements. However, singular basis functions require additional Floquet modes to achieve convergence, and the total computational time increases. In this paper, higher order hierarchical Legendre basis functions as described in [8] are used. It is shown in Fig. 1 that these higher order hierarchical Legendre basis functions can be as accurate as the singular basis functions from [7]. It is observed that these basis functions show good convergence properties when using basis functions up to 8<sup>th</sup> order. The higher order basis functions have, compared to the singular basis functions, a narrower Fourier spectrum and the need to include higher order Floquet modes can be circumvented. Additionally, the higher order basis functions are applicable to non-canonical element types, whereas the basis functions described in [7] apply only to rectangular patches or printed dipoles.

The radiation pattern of an entire reflectarray is evaluated by summing the contributions from each element as calculated from the periodic model. The contribution from each cell is determined by calculating the equivalent currents right on the reflectarray surface. On this surface, the equivalent electric and magnetic surface current densities are defined as

$$\mathbf{J}_S = \hat{n} \times \mathbf{H}, \quad (1a)$$

$$\mathbf{M}_S = -\hat{n} \times \mathbf{E}, \quad (1b)$$

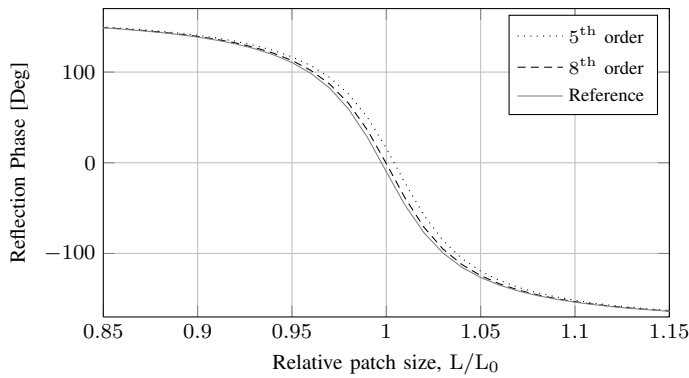


Fig. 1. Reflection phase versus the relative patch size of a rectangular microstrip patch  $L/L_0$ , where  $L_0$  is the size of a resonant patch. The order corresponds to the order of the basis functions as described in [8]. The reference is calculated using singular basis functions from [7].

where  $\mathbf{E}$  and  $\mathbf{H}$  are the aperture electric and magnetic fields, respectively, right on the surface of the reflectarray. By integrating these equivalent currents, the far field contribution from the cell is determined. This principle corresponds to Love's equivalence principle where equivalent currents are constructed on a surface enclosing the entire reflectarray and assuming the total field behind the reflectarray is zero.

### B. Extended Local Periodicity

In the local periodicity approach, the presence of neighboring elements are accounted for in an inaccurate way since these array elements are not identical. An approach was proposed in [9] to account for the mutual coupling more accurately by including the actual surrounding elements in the analysis of the array element. However, the analysis was based on a finite approach where no periodicity was applied and many elements were needed to obtain an improved result. In this paper, we propose an infinite approach where periodicity is applied on an extended unit cell, which includes the actual surrounding elements. This technique will be referred to as the Extended Local Periodicity (ELP) approach.

Each periodic cell is increased to include the nearest 8 surrounding elements in a rectangular grid. For non-edge elements the extended unit cell consists of 9 elements with the element under consideration as the central one, see Fig. 2a. The surrounding elements are the actual neighboring elements in the reflectarray. The usual SMOm is applied to the extended unit cell and the aperture field can be calculated. However, only the contribution from the element under consideration is of interest. Thus, only the aperture field confined within the dashed lines in Fig. 2a will be used. This enclosed area equals the unit cell size in the LP approach and the aperture field within this area corresponds to that calculated from the LP approach, but with contributions from actual neighboring elements. This is repeated for the next element with the extended unit cell now including the new element under consideration and its 8 neighbors. For edge/corner elements or elements with no neighbors, the element under consideration is still located at the center of the extended unit cell, but the cell

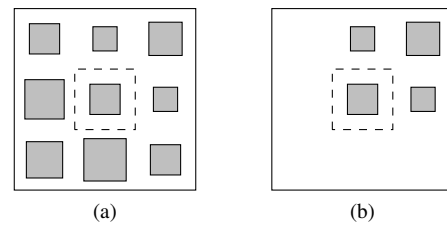


Fig. 2. Example of extended unit cells for (a) non-edge elements, (b) corners elements. The element under consideration is the central element indicated by the dashed lines.

includes only surrounding elements that are present. Fig. 2b shows an example of an extended unit cell for a corner element located at the lower left corner of a rectangular reflectarray.

In the calculation of the radiation patterns for the LP case, the aperture field at the surface of the reflectarray is approximated using only the fundamental Floquet mode from the periodic formulation [1]. This is a very accurate since the distance between the array elements is usually selected to avoid grating lobes. Consequently, all higher order Floquet modes are evanescent and only the fundamental Floquet mode contributes to the far field. In the ELP approach, the size of the unit cell is larger and more propagating modes are therefore present. In this case, additional Floquet modes must be included when the aperture field is determined.

Finally, the inclusion of the nearest surrounding neighbors in the ELP approach naturally increases the total computation time. In addition to the extra unknowns due to the increased number of basis functions, more Floquet modes are required to achieve convergence. Where the LP approach only takes a couple of seconds to calculate radiation from a realistic reflectarray, the ELP approach requires 30-45 minutes. This time consumption is too high for optimization purposes and acceleration methods are currently being investigated.

## III. APPLICATION RESULTS

### A. Reflectarray Sample

To verify the proposed technique, a printed offset reflectarray was designed. The substrate is Rogers RO4350B with a relative permittivity of  $\epsilon_r = 3.66$  and a loss tangent of  $\tan \delta = 0.0037$ . The geometrical parameters of the reflectarray are shown in Fig. 3 and summarized in Table I.

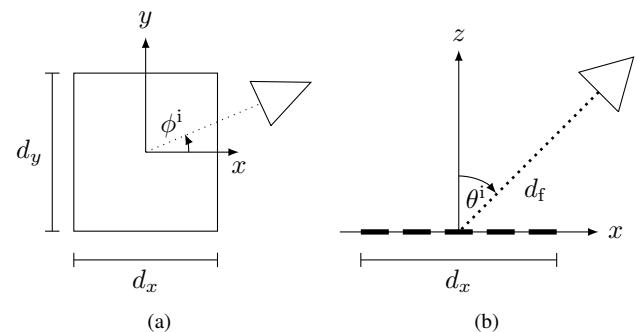


Fig. 3. Geometrical parameters of the reflectarray (a) the  $xy$ -plane (b) the  $xz$ -plane.

TABLE I  
REFLECTARRAY DATA

Reflectarray Sample	
Frequency	10 GHz
Number of elements	30×30
Reflectarray dimensions	435 × 435 mm
Substrate thickness	0.762 mm
Relative permittivity ( $\epsilon_r$ )	3.66
Loss tangent ( $\tan \delta$ )	0.0037
Feed distance	$d_f = 0.35$ m
Feed orientation	$\theta^i = 45^\circ, \phi^i = 0^\circ$

The reflectarray in Fig. 4 is designed to exaggerate the lack of periodicity with a pencil beam scanned towards  $\theta = 35^\circ$  and  $\phi = 135^\circ$  in the coordinate system shown in Fig. 3. Indeed, a strong aperiodicity is created by steering the main beam to the selected direction. The reflectarray sample is illuminated by an  $x$ -polarized polarized Potter horn with an edge illumination of approximately  $-7$  dB. The reflectarray sample and its support structures are manufactured at the Technical University of Denmark and measured at the DTU-ESA Spherical Near-Field Antenna Test Facility [10], see Fig. 5. In addition to the reflectarray measurement, the radiation of the Potter horn is also measured. From this measurement, the real incident field on the reflectarray surface is known and used in the SMoM simulations.

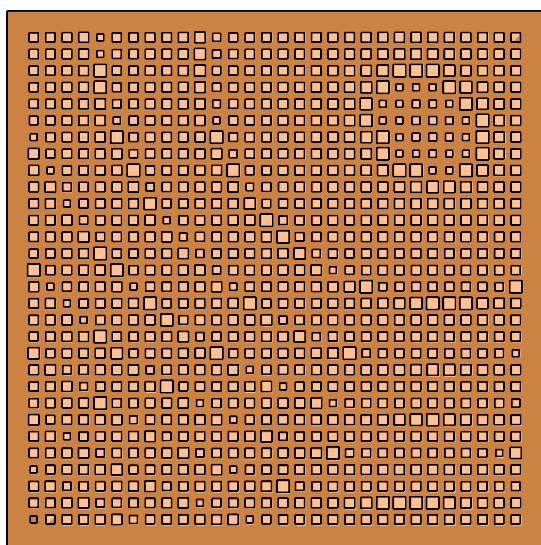


Fig. 4. Reflectarray sample designed to exaggerate the lack of periodicity with a pencil beam scanned towards  $\theta = 35^\circ$  and  $\phi = 135^\circ$  in the coordinate system shown in Fig. 3

### B. Radiation Patterns

The radiation patterns at 10 GHz obtained through measurements and simulations are compared in Fig. 6. The radiation patterns are shown in a coordinate system defined with its  $z$ -axis directed in the main beam direction. Both methods, LP and ELP, predict the maximum directivity to  $D_0 = 25.6$  dB whereas the measured is  $D_{\text{meas}} = 26.1$  dB. It is seen that

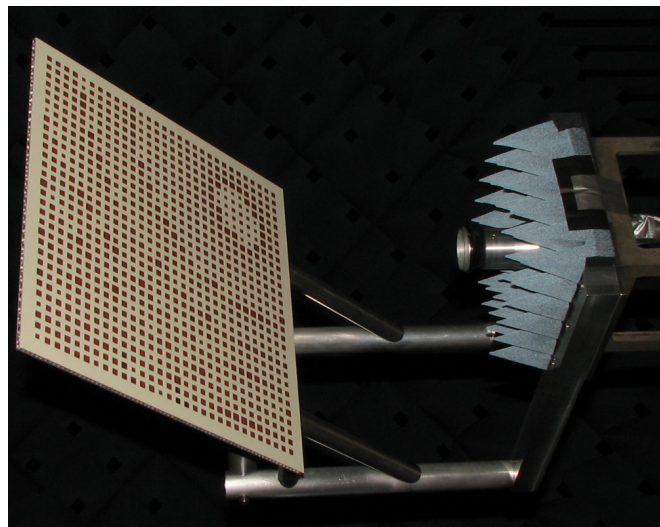


Fig. 5. Reflectarray sample in the anechoic chamber at the DTU-ESA Spherical Near-Field Antenna Test Facility.

the ELP approach is generally more accurate in the side lobe regions e.g.  $-60^\circ < \theta < -20^\circ$  in Fig. 6b and  $-40^\circ < \theta < -10^\circ$  in Fig. 6c. However, there are a few regions where the LP approach yields a better prediction, for example around  $\theta = -10^\circ$  in Fig. 6d.

Most of the scattering effects from the support structures are included in the analysis. Using the MoM add-on in GRASP [3], the scattering from the struts when radiated by the feed and the reflectarray is calculated. Due to the short distance between the feed and the reflectarray surface, the influence of the support structures are negligible in the main beam region. The scattering effects are only visible for large observation angles. This is shown in Fig. 7 where radiation patterns with and without support structures are shown. It is observed that the inclusion of the support structures greatly improves the radiation pattern for  $\theta$  values greater than  $|\theta| > 60^\circ$ .

Based on these observations it is seen that the ELP approach, as expected, indeed predicts some of the radiation features that LP cannot. However, there are still a few regions in the pattern where the LP approach is better. The authors believe that the erroneous prediction of the ELP approach in these regions is caused by the technique used to calculate the radiation pattern. In the LP approach the aperture field is calculated using only propagating Floquet modes. The aperture field is integrated over an entire period, and it has been observed that the inclusion of the evanescent Floquet modes does not contribute to the far field radiation. In the ELP approach, the aperture field is not integrated over an entire period, but truncated and integrated over a smaller region in the extended unit cell. This truncation of the aperture field may give rise to spurious radiation which can deteriorate the performance of the ELP approach. This postulate is backed up by numerical simulations where it has been shown that evanescent Floquet modes have a non-negligible effect on the radiation patterns. The issue may be circumvented by using

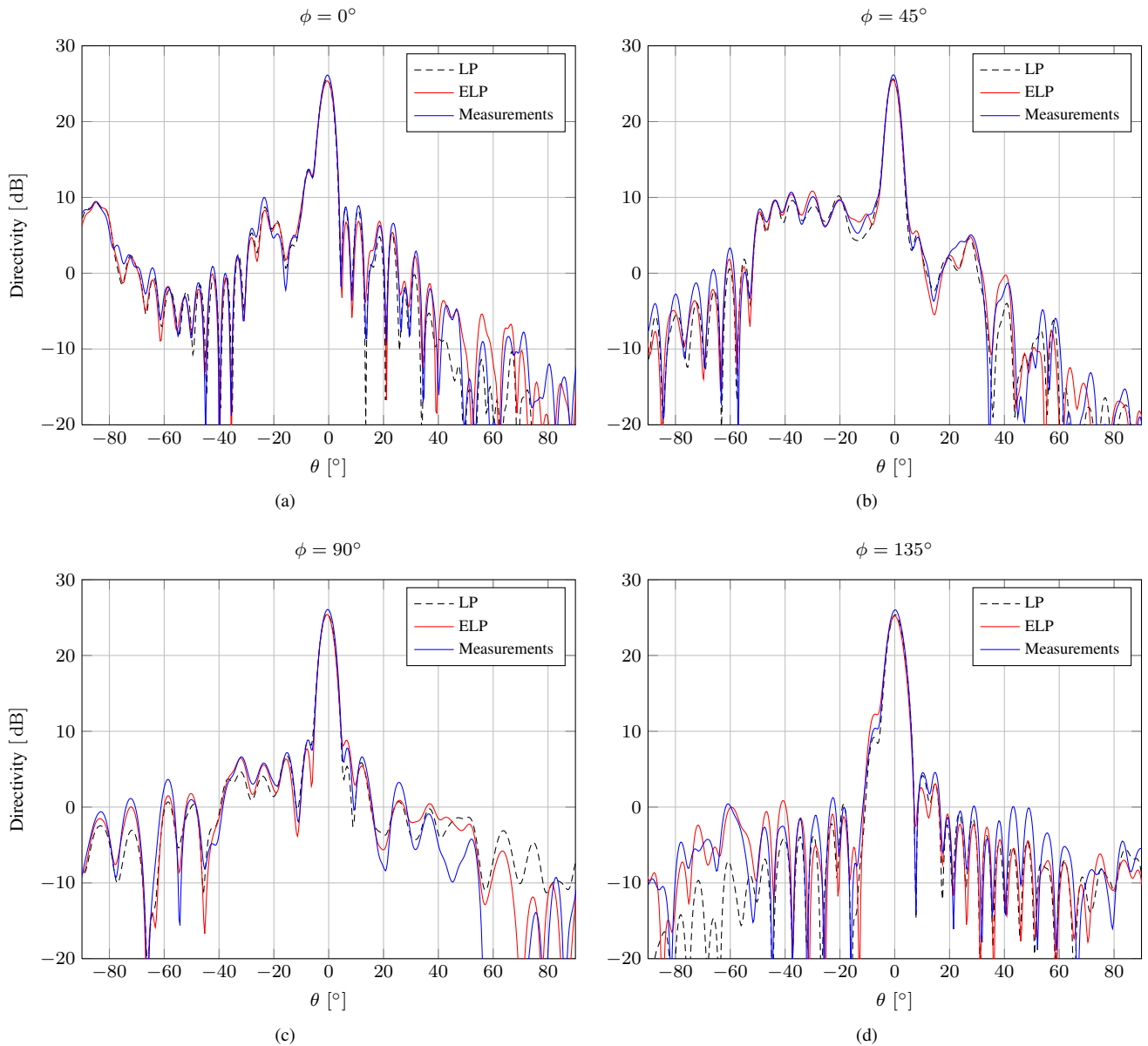


Fig. 6. Comparison of simulations using LP and ELP, and measurements.

an appropriate spatial Green's function that takes into account the presence of the ground plane and the dielectric substrate.

#### IV. CONCLUSIONS

An analysis technique that attempts to improve the modeling accuracy of printed reflectarrays is presented. The technique is referred to as the extended local periodicity (ELP) approach and is based on a periodic approach where periodicity is applied to an extended unit cell, which includes the actual elements surrounding the element under consideration. The idea behind the proposed technique is to account for the neighboring array elements in a more accurate way and thereby improve the accuracy of the analysis of printed reflectarrays.

An offset reflectarray sample designed to exaggerate the lack of periodicity in reflectarrays has been manufactured

and measured to verify the proposed technique. Comparisons of measured and simulated radiation patterns indeed show improvements in the prediction of the radiation patterns. However, there are few exceptions in the pattern where the standard LP approach is better. The authors believe that the erroneous prediction of the ELP approach in these regions is caused by the technique currently used to calculate the radiation patterns and further improvements in this area are currently being implemented.

More results will be presented at the conference.

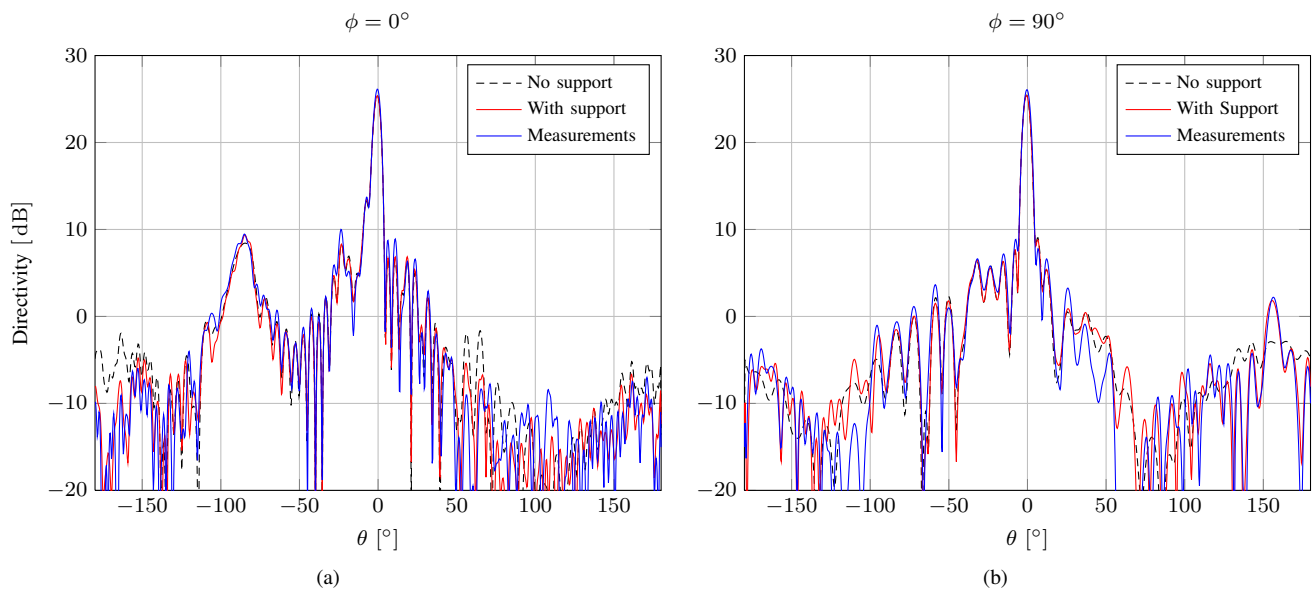


Fig. 7. Comparisons of ELP simulations with/without support structures and measurements

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