Accurate analysis of a quasi-optical network with a polarisation grid

Stig Busk Sørensen, Hans-Henrik Viskum and Knud Pontoppidan
TICRA, Læderstræde 34, 2. DK-1201 Copenhagen K

Introduction

In a quasi-optical network the beams are usually focused and guided through the system by means of curved and planar mirrors. Physical Optics (PO) is an accurate analysis tool for these elements, but is only valid for perfectly conducting surfaces and must be modified to handle other surface types such as polarisation grids, frequency selective filters and other quasi-optical elements. The usual solution is to replace the PO-currents by equivalent electric and magnetic currents calculated by means of reflection and transmission coefficients, see [1] eqn. (1)-(4). The PO-method modified in this way works well in many situations, but it has the limitation that the incident field in each point on the surface must behave locally as a plane wave, otherwise the direction of incidence is not well defined and it is not possible to assign reflection and transmission coefficients to the incident field. If the incident field comes from a point source it is straight forward to determine the direction of propagation, but for an extended source in the near field the best direction seems to be given by Poynting's vector.

In quasi-optical networks the limitations of the modified PO is often more important than in microwave antenna systems. The reason is that it is convenient to locate filters and polarisation grids close to or in the waist of a beam. The field will here have a significant radial component and the local plane wave approximation is not accurate. A more rigorous approach is to expand the incident field in a spectrum of plane waves. For each of the plane waves the direction of propagation is well-defined and equivalent currents can be calculated and added for all of the plane waves in the spectrum. The method will be illustrated by analysing one of the front-end systems of the ALMA telescope and hereafter an efficient procedure for plane-wave expansion will be described.

ALMA Band 9 front-end system

ALMA is a radio-astronomy project consisting of an array of 12-metre telescopes to be located in the high-altitude Atacama desert in Chile. Each telescope has 10 receiver front-end systems which oper-

1-4244-0878-4/07/$20.00 ©2007 IEEE

ate from 30 to 950 GHz. The Band 9 front-end system is illustrated in Figure 1, see also [2].

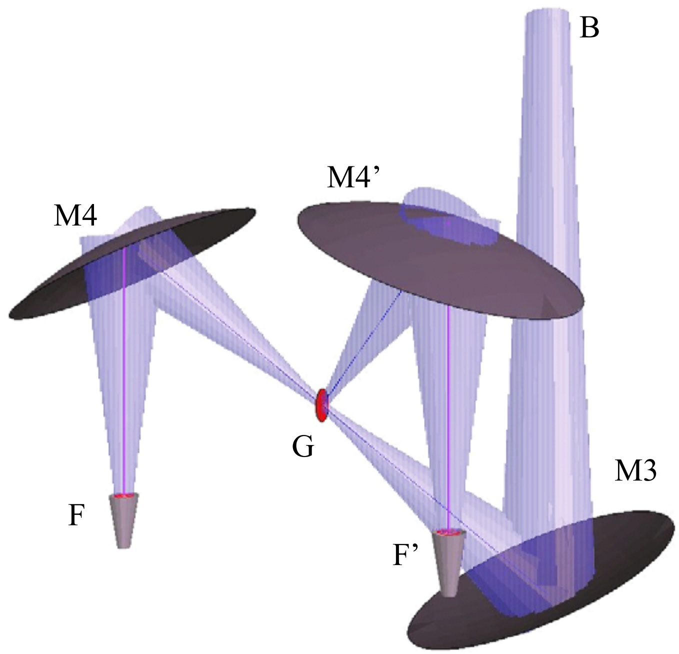

Figure 1, ALMA Band 9 front-end system.

The beam from the telescope enters the front-end system at B, is reflected by the ellipsoidal mirror M3 and split into two orthogonal linear polarisation components by the polarisation strip grid, G. The transmitted signal is guided to the feed F through reflection in M4, while M4' guides the reflected signal to F'.

The two mirrors M4 and M4' are identical, but the branch M4'-F' is rotated 125° relative to the M4 branch. From a physical point of view it is therefore expected that the cross polarisation generated by the two branches is of the same magnitude. An analysis of the beam in transmit mode at the waist B is shown in Figure 2, where the method described in [3] is applied to calculate the reflection and transmission coefficients of G.

It is seen that the method based on the local plane wave approximation predicts cross polarisation levels that differ by more than 8 dB between the two branches. The reason is that the field at the grid has a significant radial component, which contributes to the currents on the grid but is neglected by the approximation. If the rigorous plane-wave expansion is used, the correct equivalent currents are calculated and the predicted cross polar levels become equal for both branches, as expected. This illustrates the significant difference between the two methods and the importance of using the plane-wave expansion for this kind of analysis.

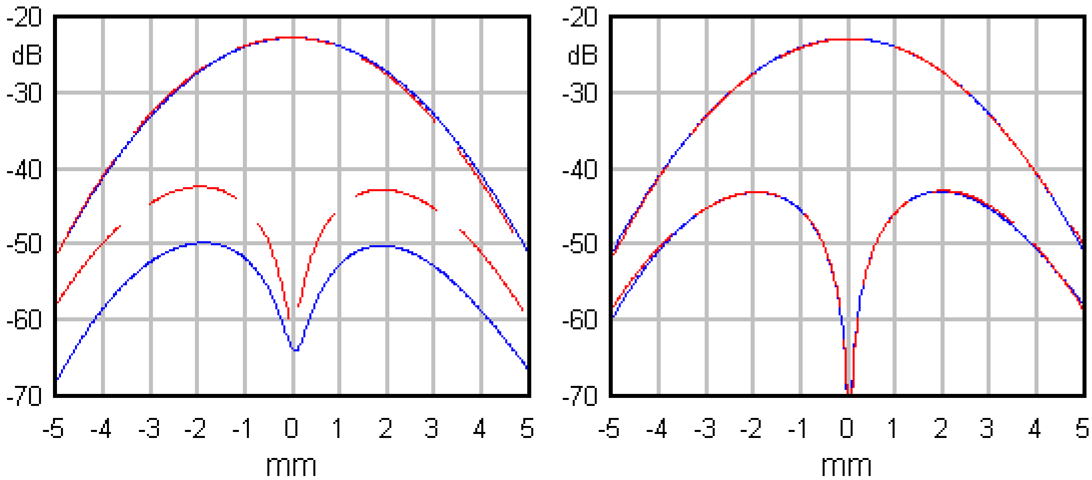

Figure 2 ALMA Band 9, 661 GHz, field in waist at B in cuts orthogonal to the plane defined by M3, M4 and F.
Left figure: local plane-wave method.
Right figure: full plane-wave expansion.
Full line (blue): F-branch. Dotted line (red): F'-branch.

Plane-wave expansion

The plane-wave expansion of an arbitrary source can be calculated efficiently by the following relations from [4]. If an xyz-coordinate system is defined such that all sources are located in the half space $z<0$ and the other half space $z>0$ is empty then it is possible to express the far field in $z>0$ as

$$\boldsymbol{E}_{far} = j\frac{e^{-jkr}}{2\pi r} k\cos\theta\, \boldsymbol{f}(k_x, k_y), \tag{1}$$

where k is the wave number and $\boldsymbol{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$, $k_x = k\sin\theta\cos\phi$, $k_y = k\sin\theta\sin\phi$, $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$ and θ, ϕ being the usual spherical coordinates. The vector function $\boldsymbol{f}$ in (1) is the spectral density. When the far field is known the function $\boldsymbol{f}$ can be found from (1) and the near field in $z>0$ is then given by

$$\boldsymbol{E}_{near}(\boldsymbol{r}) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boldsymbol{f}(k_x, k_y) e^{-j\boldsymbol{k}\cdot\boldsymbol{r}} dk_x dk_y\ . \tag{2}$$

Conversion to polar coordinates and truncation to a maximum theta angle $\theta_{\max}$ results in

$$\boldsymbol{E}_{near}(\boldsymbol{r}) = \frac{k^2}{4\pi^2}\int_0^{2\pi}\int_0^{\theta_{\max}} \boldsymbol{f} e^{-j\boldsymbol{k}\cdot\boldsymbol{r}} \cos\theta \sin\theta d\theta d\phi \,, \tag{3}$$

which can be calculated efficiently by the Gauss-Legendre rule in θ and equidistant sampling in ϕ. This gives a double sum of the form

$$\boldsymbol{E}_{near}(\boldsymbol{r}) = \sum_{t=1}^{N_\theta} \sum_{s=1}^{N_\phi(t)} \boldsymbol{q}_{st} e^{-j\boldsymbol{k}_{st}\cdot\boldsymbol{r}} \,. \tag{4}$$

Note that the number of samples in the inner sum in ϕ is a function of the index t in the outer θ-summation to avoid an unnecessary dense sampling at the pole $\theta = 0$. The relation (4) is the required plane-wave expansion corresponding to a source that radiates the far field (1). The expansion converges well close to a phase centre of the source. In the ALMA example above the computation time for the full plane-wave procedure is not significantly longer than for the local plane-wave method. As the observation point moves away from the phase centre, more and more samples would be needed in (4).

Conclusion

It is demonstrated that if significant near-field effects are present, the local plane-wave PO method for non-perfectly conducting surfaces may become inaccurate, and a full plane-wave expansion is required.

References

[1] H. P. Ip and Y. Rahmat-Samii, “Analysis and Characterization of Multilayered Reflector Antennas” IEEE Antennas Propagat. Vol. 46, No. 11 1998, pp. 1593-1605.

[2] M. Candotti, A. M. Baryshev, N. A. Trappe, R. Hesper, J. A. Murphy, J. Barkhof and W. Wild, ”Quasi-Optical Verification of the Band 9 ALMA Front-End”, ALMA Memo 544, http://www.alma.nrao.edu

[3] K. Nakamura and M. Ando, “A Full-wave analysis of offset reflector antennas with polarization grids”, IEEE Antennas Propagat. Vol. 36, No. 2 1988, pp. 164-170.

[4] R. E. Collin and F. J. Zucker, “Antenna Theory Part I”, New York: McGraw-Hill, 1969, ch. 1.