A HIGH-RESOLUTION ANTENNA DIAGNOSTICS TECHNIQUE FOR SPHERICAL NEAR-FIELD ANTENNA MEASUREMENTS

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Abstract

A new antenna diagnostics technique for spherical near-field antenna measurements, that is able to provide a high resolution of the reconstructed aperture field in excess of existing techniques, is presented. This is realized by using a transformation from the spherical wave expansion (SWE), determined from measurements, to the plane wave expansion (PWE), recovering in this way also part of the invisible region of the plane wave spectrum. By inverse Fourier transform (IFT), the aperture field on a plane very close to the antenna is determined with a resolution that exceeds the $\lambda/2$-limit, λ being the wavelength, provided by the traditional IFT of the far-field.

1 Introduction

Antenna diagnostics is a technique for detecting errors and flaws, due to manufacture as well as operation, in antennas from measurements of the radiated fields.
Planar near-field measurements or spherical near-field measurements are commonly used for this purpose, both methods presenting limits in their practical and theoretical realization. In particular, when the reconstruction of the aperture field is carried out through inverse Fourier transform of far-field data, either directly measured or derived from near-field measurements, only a spatial resolution of $\lambda/2$ is achievable, since only a limited part of the Fourier spectrum is obtainable from the far-field. Otherwise, the diagnostics can be realized through a backward-transform of planar near-field data, with the disadvantage of introducing possible errors in the invisible part of the plane wave spectrum [1]. Regarding spherical near-field measurements, even though this technique is able to realize more accurate measurements in the entire angular range, the achievable diagnostics is strongly limited by the radius of the minimum sphere that encloses the source, due to the mathematical validity of the obtained SWE [2].
In this article a new technique is derived with the purpose of exceeding the limits of existing techniques. The first step consists of obtaining the SWE coefficients from a spherical near-field measurement; second, by using and extending an existing transformation between the SWE and the PWE, the spectrum in the visible, as well as in the invisible region of the plane wave spectrum, is derived. The IFT is later applied to obtain the aperture field on a plane close to the antenna. In this manuscript the theory behind the new technique is presented and few simple test cases are provided. All theoretical results are expressed in the DTU-ESA Spherical Near-Field Antenna Test Facility notation [3] in the S.I. rationalized system with an $e^{-j\omega t}$ time dependence.

2 Theory

The SWE of the electric field $\vec{E}$, generated by a current distribution and valid in the source-free region $r > R$, is given by [3]

$$\vec{E}(\vec{r}\,) = \frac{k}{\sqrt{\eta}} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} Q_{1mn}^{(3)} \vec{F}_{1mn}^{(3)}(\vec{r}\,) + Q_{2mn}^{(3)} \vec{F}_{2mn}^{(3)}(\vec{r}\,), \tag{1}$$

where $Q_{1mn}^{(3)}$ and $Q_{2mn}^{(3)}$ are the expansion coefficients, that can be obtained from measurements, and $\vec{F}_{1mn}^{(3)}(\vec{r}\,)$ and $\vec{F}_{2mn}^{(3)}(\vec{r}\,)$ are the power-normalized spherical vector wave functions [3]. The medium intrinsic admittance is denoted by η, k is the wave number, and $\vec{r} = r\sin\theta\cos\phi\hat{x} + r\sin\theta\sin\phi\hat{y} + r\cos\theta\hat{z}$ is the position vector, with (r, θ, ϕ) being the usual spherical coordinates.
For the same field $\vec{E}$, the PWE in the spectral (k_x, k_y) domain valid for $z > z_o$, z_o being the end of the source region, is given by [4]

$$\vec{E}(\vec{r}\,) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \vec{T}(k_x, k_y) e^{j(k_x x + k_y y + k_z z)} dk_x dk_y, \tag{2}$$

where $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ is the wave propagation vector with $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$. $\vec{T}(k_x, k_y)e^{jk_z z}$ is the plane wave spectrum which can be derived by the inverse of (2) or given in terms of the source by [4]

$$\vec{T}(k_x, k_y)e^{jk_z z} = \frac{1}{4\pi k k_z}\sqrt{\frac{1}{\eta}}\vec{k} \times \left(\vec{k} \times \int_{V'} \vec{J}(\vec{r}\,')e^{-j(k_x x' + k_y y' + k_z z')}dV'\right)e^{jk_z z}. \tag{3}$$

In the following it will be shown how (1) can be transformed into (2). For this purpose, we first consider the SWE of the electric field derived by [5],

$$\vec{E}(\vec{r}\,) = \frac{1}{4\pi}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a_n^m \vec{E}_{nm}^e(\vec{r}\,) + b_n^m \vec{E}_{nm}^h(\vec{r}\,)], \tag{4}$$

where a_n^m and b_n^m are expansion coefficients, while $\vec{E}_{nm}^h(\vec{r}\,)$ and $\vec{E}_{nm}^e(\vec{r}\,)$ are expressed as

$$\vec{E}_{nm}^h(\vec{r}\,) = k h_n^1(kr)\vec{Y}_n^m(\theta, \phi), \quad \vec{E}_{nm}^e(\vec{r}\,) = \nabla \times \left[\frac{1}{jk}\vec{E}_{nm}^h(\vec{r}\,)\right], \tag{5}$$

with $h_n^1(kr)$ being the spherical Hankel function of the first kind, and $\vec{Y}_n^m(\theta, \phi)$ given by

$$\vec{Y}_n^m(\theta, \phi) = -j\frac{1}{\sqrt{2\pi}}\Big(-\frac{m}{|m|}\Big)^m\Big(\frac{\partial}{\partial\theta}\bar{P}_n^{|m|}(\cos\theta)e^{jm\phi}\hat{\phi} - \frac{1}{\sin\theta}\bar{P}_n^{|m|}(\cos\theta)jme^{jm\phi}\hat{\theta}\Big), \tag{6}$$

with $\bar{P}_n^{|m|}(\cos\theta)$ being the normalized associated Legendre function. It can be shown [6] that the two sets of spherical vector wave functions are related as

$$\vec{E}_{nm}^h(\vec{r}\,) = jk\sqrt{n(n+1)}\vec{F}_{1mn}^{(3)}(\vec{r}\,), \quad \vec{E}_{nm}^e(\vec{r}\,) = k\sqrt{n(n+1)}\vec{F}_{2mn}^{(3)}(\vec{r}\,). \tag{7}$$

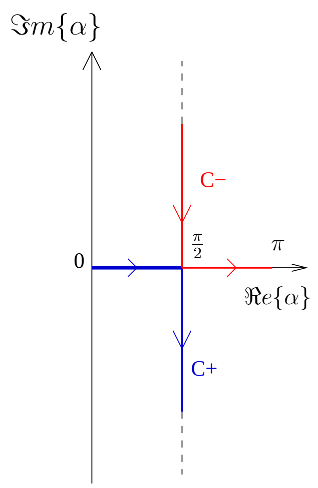


Figure 1: Countours of integration C±, for the α domain.

By comparing (1) and (4) it is thus seen that

$$Q_{1mn}^{(3)} = j\sqrt{\eta}\sqrt{n(n+1)}\frac{1}{4\pi}b_n^m, \quad Q_{2mn}^{(3)} = \sqrt{\eta}\sqrt{n(n+1)}\frac{1}{4\pi}a_n^m. \tag{8}$$

The PWE of $\vec{E}_{nm}^h(\vec{r}\,)$ and $\vec{E}_{nm}^e(\vec{r}\,)$ in the spectral (α, β) domain, valid for $|z| > z_o$, can now be introduced [5]

$$\vec{E}_{nm}^h(\vec{r}\,) = (-j)^n\frac{jk}{2\pi}\int_{-\pi}^{\pi}d\beta\int_{C\pm}\sin\alpha\vec{Y}_n^m(\alpha, \beta)e^{jk\hat{s}\cdot\vec{r}}d\alpha, \tag{9}$$

$$\vec{E}_{nm}^e(\vec{r}\,) = (-j)^n\frac{jk}{2\pi}\int_{-\pi}^{\pi}d\beta\int_{C\pm}\sin\alpha[\hat{s} \times \vec{Y}_n^m(\alpha, \beta)]e^{jk\hat{s}\cdot\vec{r}}d\alpha, \tag{10}$$

with $\hat{s} = \sin\alpha\cos\beta\hat{x} + \sin\alpha\sin\beta\hat{y} + \cos\alpha\hat{z}$, $\beta \in [-\pi, \pi]$ and $\alpha \in C_\pm$, see Figure 1. By substituting (9) and (10) into (4), and by interchanging the order of integration and summation, since the double integral is uniformly convergent, the PWE of the field in the spectral (α, β) domain, valid for every $z > z_o$, can be found as

$$\vec{E}(\vec{r}\,) = \frac{jk}{8\pi^2}\int_{-\pi}^{\pi}d\beta\int_{C_+}\sin\alpha\hat{E}(\hat{s})e^{jk\hat{s}\cdot\vec{r}}d\alpha, \tag{11}$$

where, by use of (8), the spectrum complex amplitude $\hat{E}(\hat{s})$ is given by [6]

$$\hat{E}(\hat{s}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}(-j)^n\frac{4\pi}{\sqrt{\eta}\sqrt{n(n+1)}}\Big[Q_{2mn}^{(3)}\hat{s} \times \vec{Y}_n^m(\alpha, \beta) - jQ_{1mn}^{(3)}\vec{Y}_n^m(\alpha, \beta)\Big]. \tag{12}$$

In this expression the terms $\frac{\partial}{\partial\alpha}\bar{P}_n^{|m|}(\cos\alpha)$ and $\frac{1}{\sin\alpha}\bar{P}_n^{|m|}(\cos\alpha)$ in $\vec{Y}_n^m(\alpha, \beta)$ can be calculated for every n and m by using recurrence relations involving only trigonometric functions in (α, β).

The spectrum in the spectral (α, β) domain, $\hat{E}(\hat{s})e^{jk\cos\alpha z}$, can now be translated to the (k_x, k_y) domain, to obtain $\vec{T}(k_x, k_y)e^{jk_z z}$, by using the relation [6]

$$\vec{T}(k_x, k_y)e^{jk_z z} = \frac{1}{4\pi k_z}\hat{E}(\hat{s})e^{jk\cos\alpha z}, \tag{13}$$

where every trigonometric function in α and β present in $\hat{E}(\hat{s})$ is substituted by [7]and [6]

$$\cos\alpha = \frac{k_z}{k}, \quad \sin\alpha = \sqrt{\frac{k_x^2 + k_y^2}{k^2}}, \quad \cos\beta = \frac{k_x}{k\sin\alpha}, \quad \sin\beta = \frac{k_y}{k\sin\alpha}, \quad \beta = \arctan\frac{k_y}{k_x}. \tag{14}$$

Having now obtained the PWE in the spectral (k_x, k_y) domain from the SWE (1), we can calculate the field on every plane for constant $z > z_o$ arbitrarily close to the antenna using (2). The obtainable resolution δ_x, δ_y is given by $\delta_x = \pi/k_{xmax}$ and $\delta_y = \pi/k_{ymax}$ and can thus be achieved by selecting k_{xmax} and k_{ymax} appropriately in the SWE-to-PWE transformation.

3 Test Cases

A set of Hertzian dipole configurations has been investigated [6] to verify the procedure developed in Section 2, since analytical expressions of the plane wave spectrum and the Q coefficients are available for this simple antenna model. Comparisons are carried out between the spectra in the (k_x, k_y) domain derived through (12), (13), (14) and (3), and between the fields calculated by (2), derived through (13), and the corresponding analytical field expression. The investigated test cases are:

- Case 1: A z-oriented Hertzian dipole located at the origin
- Case 2: A combination of one z-oriented Hertzian dipole displaced along z axis at $z = z_o$ and a -z-oriented displaced at $z = -z_o$ (to cancel the discontinuity of the spectrum in k_z)
- Case 3: A x-oriented Hertzian dipole located at the origin
- Case 4: A combination of one x-oriented Hertzian dipole displaced along z axis at $z = z_o$ and a -x-oriented displaced at $z = -z_o$ (to cancel the discontinuity of the spectrum in k_z).

For Case 1 an analytical verification is possible and is now considered. According to [3], the SWE involves only the term $s = 2, m = 0, n = 1$,

$$\vec{E}(\vec{r}\,) = \frac{k}{\sqrt{\eta}}Q_{201}^{(3)}\vec{F}_{201}^{(3)}(\vec{r}\,), \quad Q_{201}^{(3)} = -\frac{P}{\sqrt{6\pi}}\frac{k}{\sqrt{\eta}}, \tag{15}$$

with P being the dipole moment. The complex spectral amplitude $\hat{E}(\hat{s})$ (12) in the spectral (α, β) domain thus becomes

$$\hat{E}(\hat{s}) = -\frac{j4\pi}{\sqrt{\eta}\sqrt{2}}Q_{201}^{(3)}\hat{s} \times \vec{Y}_1^0(\alpha, \beta). \tag{16}$$

From (6) it is found that $\vec{Y}_1^0(\alpha, \beta) = j\sqrt{\frac{3}{4\pi}}\sin\alpha\hat{\beta}$ with $\hat{\beta} = -\sin\beta\hat{x} + \cos\beta\hat{y}$, so that (16) becomes

$$\hat{E}(\hat{s}) = \frac{kP}{\eta}(\cos\alpha\cos\beta\sin\alpha\hat{x} + \cos\alpha\sin\beta\sin\alpha\hat{y} - \sin^2\alpha\hat{z}). \tag{17}$$

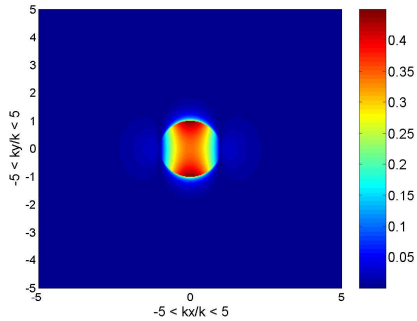
By employing the substitutions (14) and (13) in (17), the PWE for every $z > 0$ is thus obtained

$$\vec{E}(\vec{r}\,) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{P}{\eta 4\pi k}(k_x\hat{x} + k_y\hat{y} - \frac{(k_x^2 + k_y^2)}{k_z}\hat{z})e^{j(k_x x + k_y y + k_z z)}dk_x dk_y, \tag{18}$$

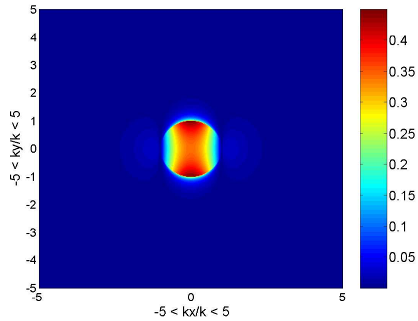
which agrees with the reference result derived from considering the dipole current $\vec{J}(\vec{r}\,) = P\delta(\vec{r}\,)\hat{z}$ in (3).

For Case 1, 2, 3 and 4 the upper limit N required for the truncation of the n-series in (12) has been analyzed. It has been found that the criterium used in the DTU-ESA facility [3], $N = [kr_o] + 10$, with r_o being the radius of the antenna minimum sphere, is not generally sufficient. Appropriate values have to be selected for every dipole case, showing a clear dependence on the antenna model and being also related to the extent of the (k_x, k_y) domain, i.e. the chosen values of k_{xmax} and k_{ymax}.

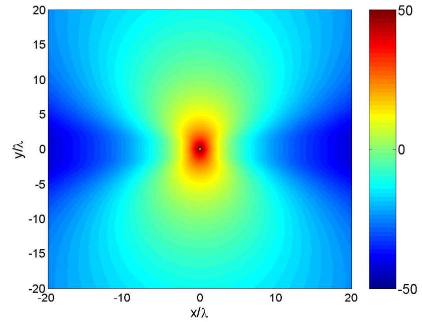
For all dipole cases, the corresponding Q coefficients are analytically derived according to [3], obtaining in general $m = -1, 0, 1$ and $n = 1, 2, .., N$. In all cases, the results, by varying z_o from 0.1λ to 0.6λ, and selecting $z = z_o + 0.3\lambda$, are extremely satisfying in the spectra as well in the fields. The results for $z_o = 0.2\lambda$, $N = [kr_o] + 14$, at $f = 2.5$GHz for Case 4, the most complicated configuration, are shown in Figure 2, for the spectrum x component in linear scale on a range of $\pm 5k$ in the (k_x, k_y) domain, and for the electric field x component in dB scale on a range of $\pm 20\lambda$ in the (x, y) domain, with a resolution of $\delta_x = \delta_y = 0.1\lambda$.



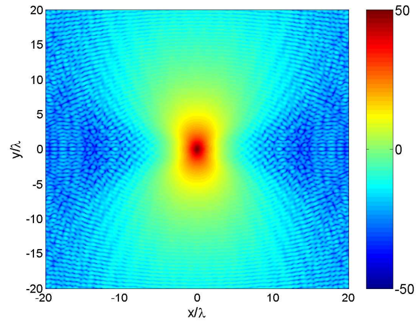
(a) Absolute value of spectrum obtained from (3).



(b) Absolute value of spectrum obtained from procedure described in Section 2.



(c) Electric field amplitude from analytic field expression.



(d) Electric field amplitude from procedure described in Section 2.

Figure 2: Numerical results for Case 4: one x-oriented dipole displaced along z axis at $z_o = 0.2\lambda$ and a -x-oriented displaced at $-z_o = -0.2\lambda$: spectrum and field (x component) are evaluated on the plane $z = 0.5\lambda$.

4 Conclusions

A new antenna diagnostics technique for spherical near-field antenna measurements has been presented. The new technique provides a high resolution in the reconstructed aperture field since the invisible region of the plane wave spectrum is taken into account. Many investigations have still to be carried out, in particular regarding the practical implications due to the denser sampling on the measurement sphere required by the high value of N, the influence of noise on the measurements accuracy, and the possible applicability to more general and real types of antennas.

References

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