# INVESTIGATIONS ON ACCURATE ANALYSIS OF MICROSTRIP REFLECTARRAYS

**M. Zhou[1,2], S. B. Sørensen[1], O. S. Kim[2], S. Pivnenko[2], and G. Toso[3]**

[1]*TICRA, Læderstræde 34, DK-1201 Copenhagen, Denmark*
[2]*Department of Electrical Engineering, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark*
[3]*ESA-ESTEC, Keplerlann 1, PB 299, 2200 AG Noordwijk, The Netherlands*

## ABSTRACT

An investigation on accurate analysis of microstrip reflectarrays is presented. Sources of error in reflectarray analysis are examined and solutions to these issues are proposed. The focus is on two sources of error, namely the determination of the equivalent currents to calculate the radiation pattern, and the inaccurate mutual coupling between array elements due to the lack of periodicity. To serve as reference, two offset reflectarray antennas have been designed, manufactured and measured at the DTU-ESA Spherical Near-Field Antenna Test Facility. Comparisons of simulated and measured data are presented to verify and demonstrate the improved results using the proposed solutions.

Key words: Microstrip reflectarrays; accurate antenna analysis; method of moments (MoM), antenna radiation pattern, horn antennas, measurements.

## 1. INTRODUCTION

Microstrip reflectarrays are becoming viable alternatives to reflector antennas for satellite applications and have been the subject of increasing research interest [1–4]. To obtain high-gain performance for satellite applications, the electric size of reflectarrays is usually very large, and therefore an efficient and accurate analysis is a challenging task. The commonly adopted analysis method is based on the spectral domain Method of Moments (SDMoM) assuming local periodicity (LP), that is, an individual array element is embedded in an infinite array of identical elements [5]. This approach has been demonstrated to be efficient for reflectarrays made of varying-sized patches, and many advanced reflectarrays have been designed using this technique [3]. However, reflectarrays are aperiodic by nature and the local periodicity assumption gives rise to discrepancies between simulated and measured radiation patterns. Efficient full-wave simulation techniques have been applied on entire reflectarrays for accurate determination of the currents on the array elements [6–9]. However, discrepancies between simulations and measurements can still be observed, and the increase in computation time makes the methods unaffordable for optimization processes. For space applications, where the accuracy demands are high, an efficient and accurate analysis method is important to precisely determine the radiation properties of reflectarrays, and it is essential for optimization purposes. The objective of this work is to understand the sources of error in reflectarray analysis and to propose solutions to these.

This paper is organized as follows. Section 2 describes some of the sources of error in reflectarray analysis. The benchmark antennas are described in Section 3. In Section 4, simulations are compared with measurements, and conclusions are given in Section 5.

## 2. SOURCES OF ERROR

In reflectarray analysis, several factors can give rise to errors, e.g. the representation of the incident field, the choice of basis functions, the technique used to calculate the radiation patterns, the lack of periodicity, the truncation of the ground plane, scattering from the support structures etc. All these factors should be accounted for correctly in order to obtain an accurate analysis.

### 2.1. Representation of the Incident Field

In the SDMoM computations, each array element is assumed to be illuminated by a locally plane wave. The pattern of the feed is usually approximated by a far field model using a $\cos^q(\theta)$ function or a Gaussian beam [1, Sec. 3.8], and is used to compute the polarization of the incident plane wave on each array element. This approximation is usually inaccurate due to the idealized feed pattern. The analysis accuracy can be improved by using the real pattern of the feed obtained by either measurements [10] or commercially available simulation tools e.g. CHAMP [11]. However, the assumption of plane wave incidence in the SDMoM is only valid if the reflectarray surface is located sufficiently far away from the feed. Alternatively, a plane wave expansion of the feed radiation over the reflectarray surface can be computed. The SDMoM analysis is then performed for each plane wave and subsequently added to yield the final result. In this approach, the representation of the incident field is exact, however at the cost of computation time. For the reflectarray antennas to be described in Section 3, it was found that the assumption of plane wave incidence is a sufficient representation of the incident field.

## 2.2. Choice of Basis Functions

The choice of basis functions in the SDMoM computations is important. Roof-tops and entire domain trigonometric basis functions are popular choices. However, for resonant microstrip array elements, the convergence of the SDMoM solution becomes poor and in certain cases convergence is never achieved using these basis functions. Consequently, singular basis functions with correct edge conditions are required for accurate characterization of the array elements [12]. However, singular basis functions require additional Floquet modes to achieve convergence, thus increasing the total computation time. In this paper, higher order hierarchical Legendre basis functions as described in [13] are used. These basis functions can be applied to any non-canonical element types, and at the same time yield results that are identical to those obtained with singular basis functions using less computation time [14].

## 2.3. Aperiodicity

Reflectarrays are inherently aperiodic due to the need to compensate for the spatial phase delay from the feed. Thus the periodicity assumption in the LP approach can give inaccurate results.

One way to reduce these errors is to use the "Surrounded Element Approach" (SEA) proposed in [15]. The analysis in SEA is based on a finite approach where no periodicity is applied. It includes the actual neighboring elements that surrounds the element under consideration, thus accounting for the mutual coupling more accurately. In [15], the analysis was based on a FDTD implementation assuming plane wave incidence, and many neighboring elements were required to obtain an improved result. Even though the reported computation time were in terms of hours, we find this technique interesting and promising. Therefore, an integral equation (IE) formulation using a spatial dyadic Green's function (DGF) [16] is currently being implemented. Using an IE formulation, the plane wave incidence assumption is avoided and the total computation time may be reduced. Preliminary results indicate that the SEA is especially accurate in predicting the radiation pattern of the cross-polarization component.

Another approach, which is a combination of the LP approach and the SEA, is introduced in [14], the "Extended Local Periodicity" (ELP) approach. Similar to the LP approach, the ELP approach is also based on periodicity, but the periodicity is applied on an extended unit cell which includes the actual 8 neighboring elements that surround the element under consideration. SDMoM is applied to the extended unit cell and the unknown currents on the element under consideration is determined. This is repeated for the next element with the extended unit cell now including the new element under consideration and its 8 neighbors. The inclusion of the nearest surrounding neighbors increases the total computation time, since the number of basis functions is larger and additional Floquet modes are required. While an analysis of a realistic reflectarray takes a couple of seconds using the LP approach, the ELP approach requires 40-60 minutes. This increase is significant and must be decreased if the approach is to be used for optimization purposes. Acceleration techniques on this matter can be found in the literature [17, 18] and are currently being investigated.

Recently, there have been interests in completely aperiodic reflectarrays that aim to exploit all available degrees of freedom of the array elements e.g. element positions and orientations [19, 20]. The array elements are located in irregular grids and the periodic assumption in the LP approach is inaccurate. It is expected that the SEA and the ELP approach are better candidates for analysis of such reflectarrays.

## 2.4. Truncation Effects

Due to the periodicity assumption in the LP and ELP approaches, the truncation of the ground plane is not taken into account in the determination of the currents on the array elements. Nor is it accounted for in the IE formulations since the spatial DGF assumes infinite ground plane. An efficient and accurate solution to this issue should be investigated.

## 2.5. Radiation Pattern Calculations

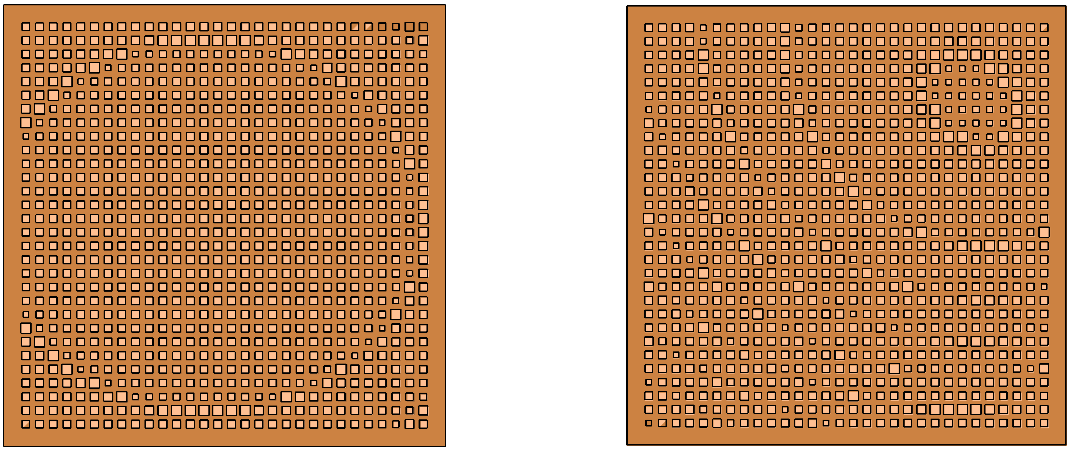
In the literature on accurate analysis of reflectarrays, the main focus has been on the determination of the currents on the array elements, and the technique used to calculate the radiation pattern has received less attention. However, the latter is equally important.

The conventional technique, which will be referred to as technique I, to calculate the radiation pattern is to invoke the field equivalence principle to calculate equivalent electric and magnetic surface currents in the plane of the array elements [21, 22]. The equivalent electric and magnetic currents are defined as

$$\boldsymbol{J}_{\mathrm{S}} = \sum_i \boldsymbol{J}_i = \sum_i \hat{z} \times \boldsymbol{H}_i, \quad (1)$$

$$\boldsymbol{M}_{\mathrm{S}} = \sum_i \boldsymbol{M}_i = -\sum_i \hat{z} \times \boldsymbol{E}_i, \quad (2)$$

where index $i$ runs over all array elements. The electric and magnetic fields, $\boldsymbol{E}_i$ and $\boldsymbol{H}_i$, respectively, are approximated using the fundamental Floquet mode from the discrete spectrum in the SDMoM simulations. By integrating the equivalent currents, the far field radiation pattern can be determined. There are several disadvantages with this technique. Firstly, the ground plane in reflectarrays is often extended at the edges for practical reasons [22, 23]. The extended ground plane contributes to the far field radiation and is not accounted for in this technique. This can be circumvented by placing unit cells with no elements at the edges to cover the area of the extended ground plane. However, this procedure is rather impractical and not suitable for commercial codes. Secondly, equivalent currents are discontinuous at cell boundaries.



(a) Antenna I

(b) Antenna II

*Figure 1. Reflectarray mask for the antenna I and II.*

These discontinuities can contribute to phase and amplitude errors in the equivalent currents, thus resulting in an erroneous relation between the equivalent electric and magnetic currents. As a result, the radiation in the back hemisphere can be wrong [24].

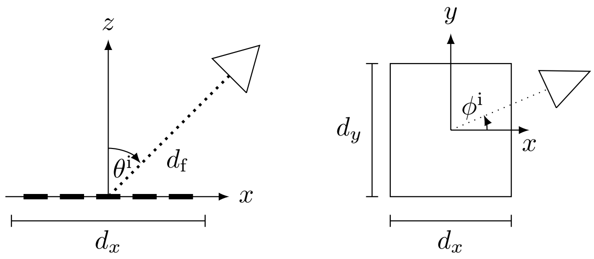
To avoid the issues associated with technique I, a novel technique as described in [24] can be used. This technique will be referred to as technique II and it also utilizes the field equivalence principle. Instead of calculating the equivalent currents using the discrete spectrum from the SDMoM simulations, a continuous spectrum is employed. In this technique, the extended ground plane is automatically accounted for. Additionally, no discontinuities are created in the equivalent currents, and the equivalent electric and magnetic currents are correctly related through the continuous spectrum, thus enabling an accurate calculation of the radiation pattern on the entire far field sphere.

## 3. BENCHMARK ANTENNAS

To serve as benchmark cases, two offset microstrip reflectarray antennas were designed. The designs are focused on the aforementioned sources of error and are intended to exaggerate these sources such that they can be separated. The radiation patterns of the antennas were measured at the DTU-ESA Spherical Near-Field Antenna Test Facility [25] and serve as reference solutions.

The reflectarray in Figure 1a is designed to exaggerate the truncation effects. To do so, a strong edge illumination is required and the aperiodicity effects must be reduced. To this end, a smooth patch variation is obtained by reducing the spatial phase delay by placing the feed far from the reflectarray surface, and by directing a pencil beam towards the specular direction.

The reflectarray in Figure 1b is designed to exaggerate the aperiodicity in reflectarrays by having a pencil beam towards $\theta = 35°$ and $\phi = 135°$ in the coordinate system




(a)

(b)

*Figure 2. Geometrical parameters of the reflectarray (a) the $xz$-plane (b) the $xy$-plane.*

in Figure 2. To ensure that the truncation effects are negligible, the feed is located close to the reflectarray surface such that a low edge illumination can be achieved with a high gain feed.

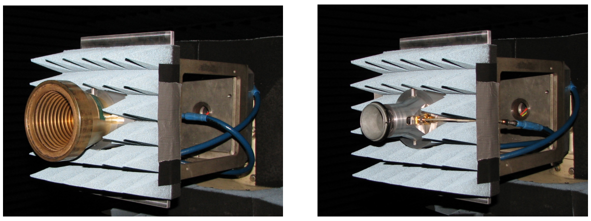
For both antennas, the substrate is Rogers RO4350B with a relative permittivity of $\epsilon_\mathrm{r} = 3.66$ and a loss tangent of $\tan\delta = 0.0037$. The ground plane and substrate for both antennas are slightly extended at the edges as seen in Figure 1. The geometrical parameters of the reflectarrays are shown in Figure 2 and summarized in Table 1.

Two horn antennas were used as feed, a corrugated horn (Figure 3a), and a Potter horn (Figure 3b). Each horn was used on both antennas giving a total of 4 different reflectarray configurations. The corrugated horn has at $10\,\mathrm{GHz}$ a taper of $-17.5\,\mathrm{dB}$ at $30°$ whereas the Potter horn has a taper of $-7\,\mathrm{dB}$ at $30°$.

The reflectarrays and their support structures were manufactured at the Technical University of Denmark. For the measurements, see Figure 4, the estimated $1\sigma$ uncertainty for the peak directivity is $0.07\,\mathrm{dB}$. In addition to the reflectarray measurements, the feed horns were also measured and the measured data was used to represent the incident field on the reflectarray surface in the analysis.

*Table 1. Benchmark Antenna Data*

| | |
|---|---|
| Frequency | 10 GHz |
| Number of elements | 30×30 |
| Reflectarray dimensions | 435 mm × 435 mm |
| Substrate thickness | 0.762 mm |
| Relative permittivity ($\epsilon_r$) | 3.66 |
| Loss tangent ($\tan\delta$) | 0.0037 |
| **Antenna I** | |
| Feed distance: | $d_f = 0.6$ m |
| Feed orientation: | $\theta^i = 30°$, $\phi^i = 0°$ |
| Main beam direction: | $\theta = -30°$, $\phi = 0°$ |
| **Antenna II** | |
| Feed distance: | $d_f = 0.35$ m |
| Feed orientation: | $\theta^i = 45°$, $\phi^i = 0°$ |
| Main beam direction: | $\theta = 35°$, $\phi = 135°$ |



(a) (b)

*Figure 3. The two horn antennas used in the benchmark cases, (a) the corrugated horn, (b) the Potter horn.*

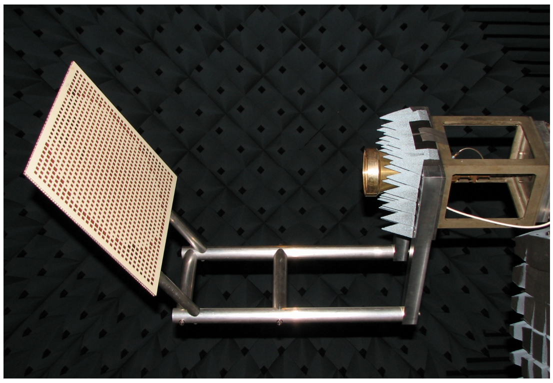
These reflectarray antennas and their associated radiation patterns constitute a scientific contribution in itself, since the results currently available in the literature are not completely specified in terms of geometrical parameters or feed radiation pattern, or the accuracy of the measurements. In addition, the available results are for complex full-size reflectarrays where many sources of errors are present at the same time and cannot be distinguished.

## 4. SIMULATIONS VS. MEASUREMENTS

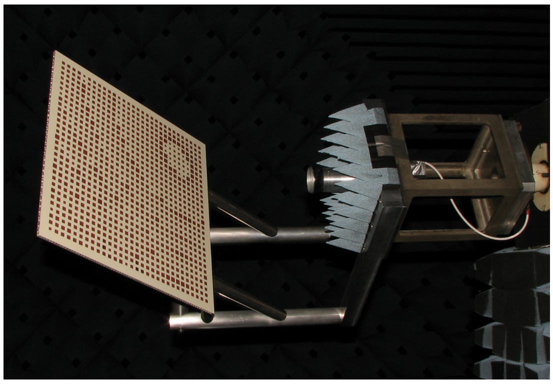
Radiation patterns obtained at 10 GHz by measurements and simulations are presented in this section. To account for the presence of the support structures, the scattering from the struts is included in the analysis using the MoM add-on in GRASP [26]. All radiation patterns are depicted in a coordinate system defined with the $z$-axis directed towards the main beam direction.

### 4.1. Radiation Pattern Calculations

To demonstrate the accuracy of the aforementioned techniques to calculate the radiation pattern, technique I and II are used to compute the radiation patterns for antennas I and II, and compared with measurements in Figure 5 and Figure 6. For these results, the currents on the array elements are determined using the LP approach. To illustrate the effect of the extended substrate, this contribution is not included in technique I.



(a) Antenna I with the corrugated horn



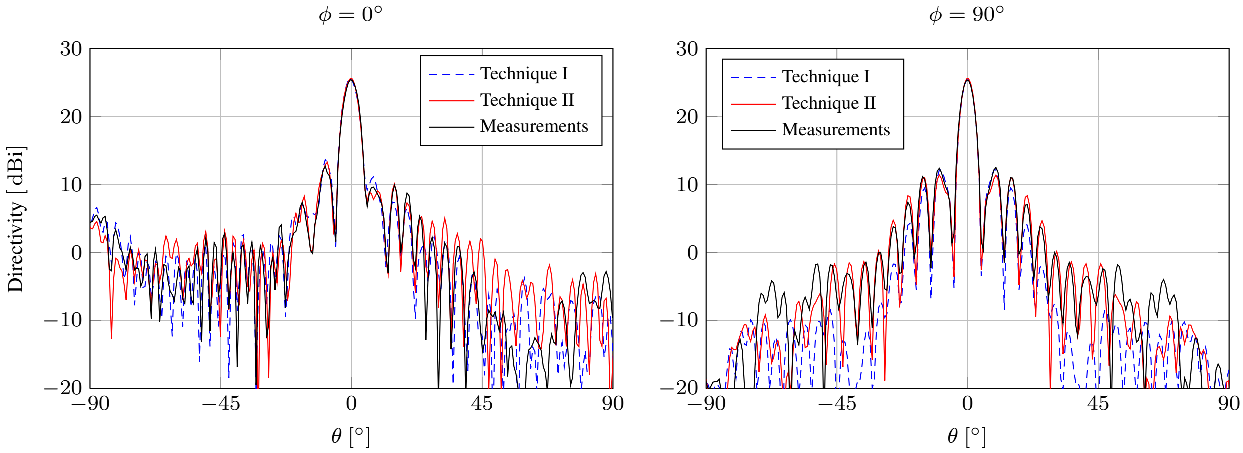
(b) Antenna II with the Potter horn

*Figure 4. Benchmark antennas at the DTU-ESA Spherical Near-Field Antenna Test Facility.*

The far field co-polar radiation patterns for antenna I are shown in Figure 5. The Potter horn is used in this case to give a strong edge illumination, $-7$ to $-1$ dB. It is seen that the improvement using technique II is clear, especially in Figure 5b where the simulated and measured pattern practically coincide. The discrepancies observed in Figure 5a around $\theta = 45°$ are due to the blockage of the measurement tower.

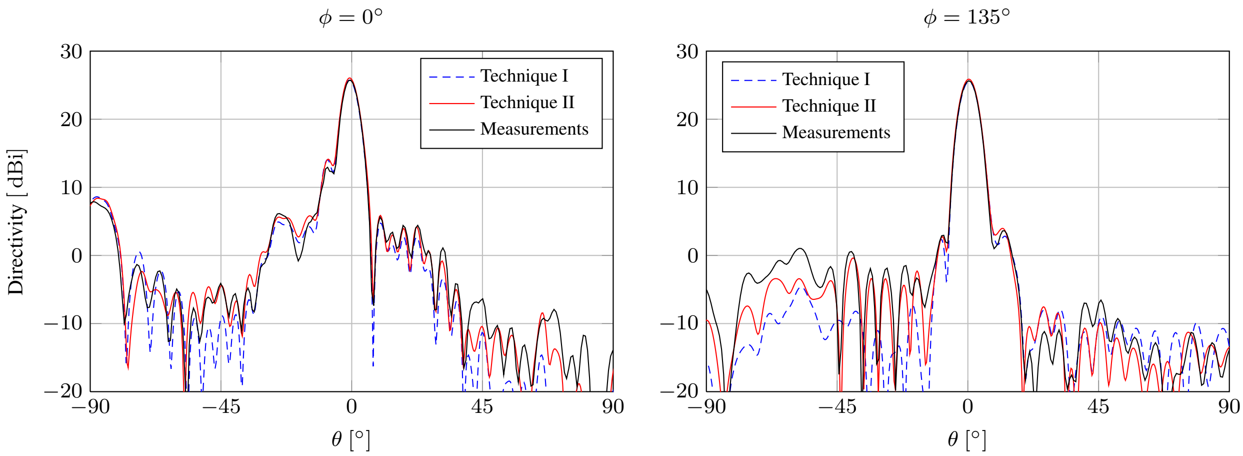
The far field co-polar radiation patterns for antenna II are depicted in Figure 6. A low edge illumination of approximately $-25$ to $-12$ dB is achieved in this case using the corrugated horn as feed. Even in this case where the edges are weakly illuminated, the enhancement of the analysis accuracy is apparent. The side lobes are much better predicted using technique II compared to technique I. The discrepancies are mainly attributed to the strong aperiodicity.

The improvements in Figure 5 and Figure 6 using technique II are due to the inclusion of the extended ground plane. It is shown in [24] that a similar accuracy in the forward hemisphere can be achieved with technique I if empty unit cells are placed at the edges to cover the area extended by the substrate. Since very good agreements can be achieved when the aperiodicity effects are minimized, it can be concluded that the error introduced by the truncation of the ground plane is of small importance




(a) (b)

*Figure 5. Comparison of radiation pattern calculated using technique I and II with measurements for antenna I.*




(a) (b)

*Figure 6. Comparison of radiation pattern calculated using technique I and II with measurements for antenna II.*

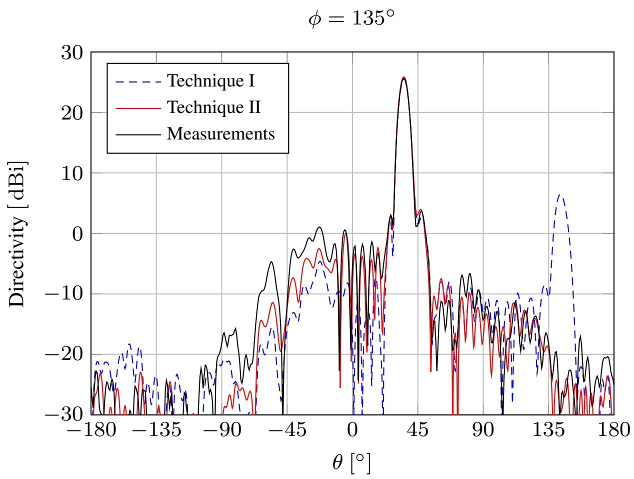



*Figure 7. The results from Fig. 6b shown also in the back hemisphere.*

for the determination of the currents on the array elements, but has to be accounted for in the calculation of the far field radiation pattern.

With respect to the radiation in the back hemisphere, technique I can yield wrong results as shown in Figure 7. In this figure, the results from Figure 6b are shown also in the back hemisphere. It is seen that in the direction of the main beam's image around $\theta = 145°$, an erroneous beam is predicted by technique I. This is caused by the incorrect relation between the equivalent electric and magnetic currents in technique I as previously mentioned. This issue is avoided using technique II since the equivalent currents are correctly related to each other, thus yielding good results in the entire far field sphere.

## 4.2. Aperiodicity

To illustrate the aperiodicity effects, results for antenna II computed using the LP and ELP approaches are com-

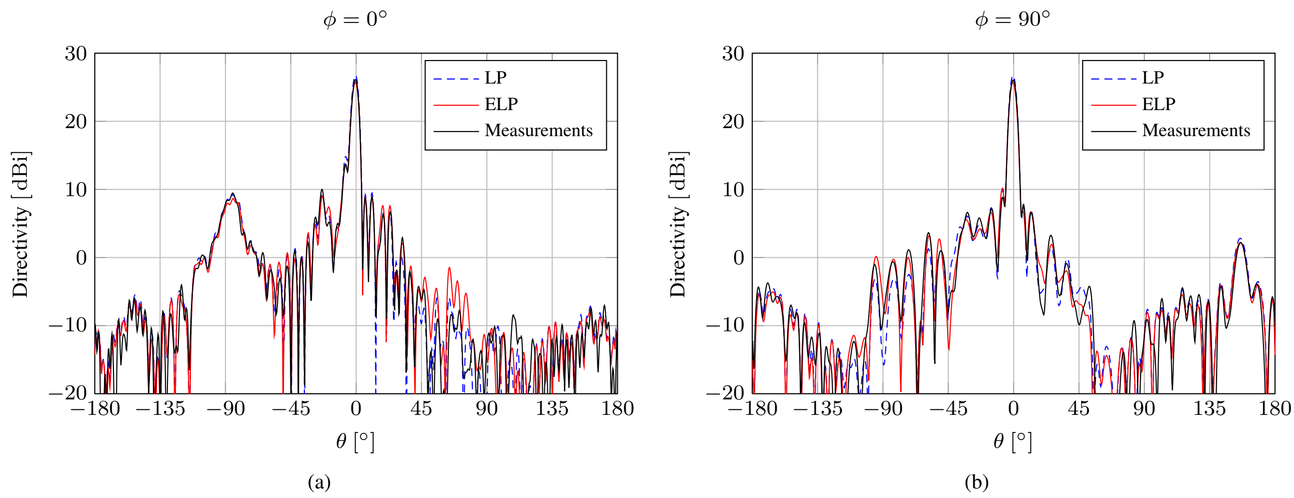



*Figure 8. Comparison of simulations using LP and ELP, and measurements for antenna II. The Potter horn is used as feed.*

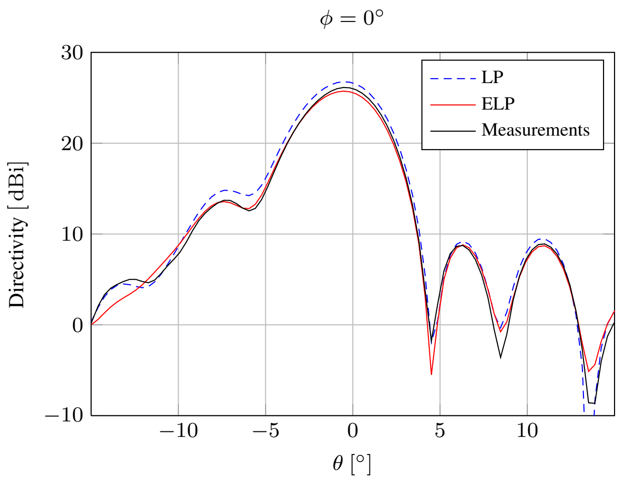



*Figure 9. Close up of the comparison shown in Fig. 8a.*

pared with the measurements and shown in Figure 8. Technique II is used to calculate the radiation patterns, and the Potter horn is used as feed yielding an edge illumination of approximately $-20$ to $-5$ dB.

It is seen that the overall accuracy over the entire far field sphere is good, also for the LP case. This is due to the technique used to calculate the radiation pattern and the inclusion of the radiation from the support structures. However, it is observed that the ELP approach is generally more accurate e.g. $-90 < \theta < -40$ in Figure 8b. To see the improvements more clearly, a close up of the radiation pattern from Figure 8a is depicted in Figure 9. It is apparent that the accuracy is enhanced using the ELP approach. The peak directivity is measured to $D_{\text{meas}} = 26.1$ dBi. The LP approach predicts $D_{\text{meas}} = 26.8$ dBi whereas ELP yields an improved value of $D_{\text{meas}} = 25.9$ dBi. For the far field cross-polar radiation patterns, the results obtained using the LP and ELP approaches are rather similar and improvements were not observed.

## 5. CONCLUSIONS

An investigation on accurate analysis of microstrip reflectarrays has been carried out and is presented in this paper. Several sources of error in reflectarray analysis have been examined, e.g. the representation of the incident field, the choice of basis functions, aperiodicity, and truncation effects, as well as the technique used to calculate the radiation pattern. To serve as benchmark cases, two offset reflectarray antennas have been designed, manufactured and measured at the DTU-ESA Spherical Near-Field Antenna Test Facility. The two designs aim to exaggerate the errors introduced by the truncation of the ground plane, and the lack of periodicity, respectively. Comparisons of simulated and measured radiation patterns show that the errors introduced by the truncation of the ground plane are of small importance for the determination of the currents on the array elements and can thus be neglected. The errors introduced by the aperiodicity on the other hand can be significant and the conventional local periodicity assumption can be inaccurate. To circumvent this, the extended local periodicity approach can be utilized. In addition, the comparisons shown that the choice of the technique to calculate the radiation pattern is very important with respect to the analysis accuracy. The finite substrate and ground plane size of the reflectarray must be taken into account and techniques that neglect this give inaccurate results.

The results presented in this paper show that accurate analysis of reflectarrays can be achieved if the sources of error are treated correctly. However, several challenges remain and are presently being investigated.

## ACKNOWLEDGMENTS

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