Analysis of reflectarrays using field symmetries

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Abstract-- A new method to analyze reflectarrays with rectangular patches is proposed. While the standard procedure has been to calculate the electric currents on the patches using e.g. a periodic Green's function or a Method of Moments approach, the present method calculates the magnetic currents in the gaps between the patches and subsequently determines the radiated field using Love's equivalence theorem. Since the complete analysis can be expressed on closed, analytical form, the method is extremely fast and makes no implicit assumptions about periodicity.

Index Terms- Reflectarray

1. INTRODUCTION

This paper presents an alternative method to analyze reflectarrays with rectangular patches. The objective is to determine the electric fields in the gaps between the patches. These fields can be converted into magnetic currents, and the radiated field can then be calculated using Love's equivalence theorem, i.e. the magnetic currents are placed on a perfectly electrically conducting (PEC) surface. First the PEC surface is removed and the magnetic currents are doubled to account for the mirror images, and the far field is calculated using the free space Green's function. Subsequently the reflection from the PEC surface is added to obtain the total field.
To calculate the electric field in the gaps, the incident field is first separated into two components, one that only excites a field in the x directed gaps, and one that only excites a field in the y directed gaps. The paper is organized as follows: first a reflectarray with identical square patches is analyzed for an incident plane wave to establish the phase curve for the reflected wave.
As a byproduct the electric field in the gaps are also calculated. This is done for boresight incidence, for E-plane scan and finally for arbitrary incidence.
Finally an actual reflectarray is analyzed, assuming that the incident field from the feed is locally a plane wave over each patch. All magnetic currents are calculated and the radiated field is calculated.

2. BORESIGHT ANALYSIS

Fig.1 shows a reflectarray lying in the x-y plane with a plane wave incident from the z axis. The field is polarized along x, hence, since the gaps are narrow compared to the wavelength, the field will not be able to penetrate the x directed gaps. We can therefore replace the array of patches with an array of y directed strips, as shown in Fig. 1.

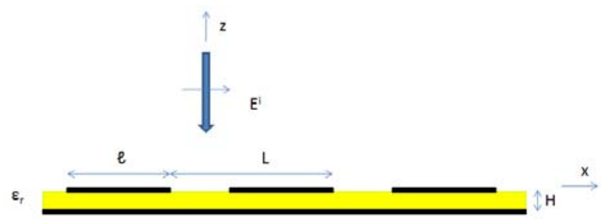

Figure 1. Strip array model of reflectarray.

Clearly the electric field can penetrate the y directed gaps, and due to symmetries, the field that penetrate two neighboring gaps will launch waves traveling in opposite directions, vertically polarized, with opposite polarization. These two waves will have identical phase when they meet at the centre of the strip, and will therefore exactly cancel at this point. This means that under each strip, the vertical electrical field will be exactly zero at the centre, and we can therefore insert a perfectly conducting wall at these points without changing the electromagnetic field. This is shown in Fig. 2.

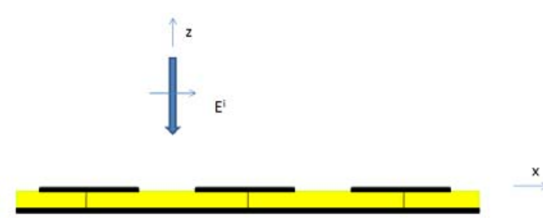

Figure 2. Strip array with short circuits inserted.

The effect of these walls is to decouple the field propagating under the strips. We can now consider the field penetrating one gap separately. This field will propagate into two shunts, one to the left and one to the right. Since the thickness of the dielectric is very small in wavelengths, it is immaterial whether the field in the shunts propagate horizontally or vertically. We can therefore replace the two horizontal shunts with two vertical shunts, and since they are equally long, we can merge them into one shunt of width 2H, where H is the thickness of the dielectric. This is shown in Fig. 3.

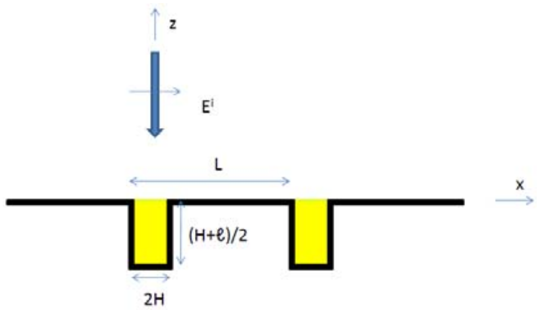

Figure 3. Corrugated surface model.

Effectively we have now replaced an array of strips with a corrugated surface. We assume that the field in the corrugation, z<0, is a standing wave (excluding any losses in the dielectric) and match this wave to the incident and reflected plane waves for z>0 across the boundary at z=0. Suppose the complex amplitudes of the incident and reflected waves are one and β. We then find

$$\beta = -\frac{T^2}{|T|^2}, T = \frac{L}{2H} + i\frac{k}{k_r}\tan\left(k_r\frac{\ell + H}{2}\right) \quad (1)$$

where k and k_r are the propagation constants in free space and in the dielectric.
To modify the procedure for an incident wave scanned in the H plane is trivial, essentially requiring only that the propagation vector is decomposed into a vertical component and a component along the y axis.

3. E-PLANE SCAN

Scanning the incident wave in the E-plane requires a little more consideration. We can still replace the patch array with a strip array, but the perfect symmetry is no longer present. We shall therefore introduce the Ansatz that the waves excited below the strips are still symmetrical in amplitude, but obviously have different phases, depending on the scan angle. It is, however, a trivial geometric exercise to determine the position where the two waves will arrive with identical phases, and therefore cancel each other. As shown in Fig. 4 the conducting walls will therefore be offset from the centre.

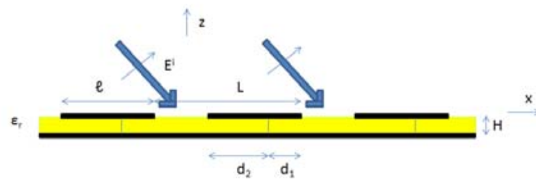

Figure 4 Strip array model of reflectarray.

When the model is modified into a corrugated surface, we can no longer merge the two shunts, since they are of different length. As shown in Fig. 5, we must therefore model the corrugations as bifurcated waveguides.
Solving the scattering problem is only slightly more complicated, compared to the boresight case.

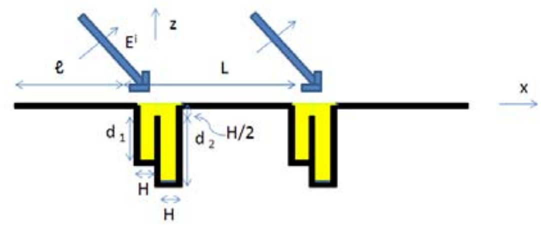

Figure 5. Corrugated surface model.

We still assume a standing wave in the corrugation, but only for -H/2<z<0. At z=-H/2 we impose a condition derived from considering the two parallel shunts as a bifurcated waveguide, and proceed to match the fields at z=0. We now find

$$\beta = -\frac{T^2}{|T|^2}, \ T = \frac{L}{2H} + i\frac{k\cos\theta}{k_r}\tan\left(\frac{k_r H + \Psi}{2}\right) \quad (2)$$

where Ψ describes the phase of the reflection coefficient of the bifurcated waveguide at z=-H/2.

4. ARBITRARY INCIDENCE

We now consider a plane wave incident from the direction (θ, φ) in the coordinate system (x_G, y_G, z_G), where the patches are aligned along (x_G, y_G). The propagation vector of the incident field is $\hat{\underline{k}}^i$, so we now define the two vector systems ($\underline{\hat{x}}^{i}{}_{v1}, \underline{\hat{y}}^{i}{}_{v1}, \underline{\hat{k}}^{i}$) and ($\underline{\hat{x}}^{i}{}_{v2}, \underline{\hat{y}}^{i}{}_{v2}, \underline{\hat{k}}^{i}$) as

$$\underline{\hat{y}}^{i}{}_{v1} = \frac{\underline{\hat{y}}_G - \underline{\hat{k}}^{i} \bullet \underline{\hat{y}}_G \underline{\hat{k}}^{i}}{| \underline{\hat{y}}_G - \underline{\hat{k}}^{i} \bullet \underline{\hat{y}}_G \underline{\hat{k}}^{i} |}, \; \underline{\hat{x}}^{i}{}_{v1} = \underline{\hat{y}}^{i}{}_{v1} \times \underline{\hat{k}}^{i} \qquad (3)$$

and

$$\underline{\hat{y}}^{i}{}_{v2} = \frac{\underline{\hat{x}}_G - \underline{\hat{k}}^{i} \bullet \underline{\hat{x}}_G \underline{\hat{k}}^{i}}{| \underline{\hat{x}}_G - \underline{\hat{k}}^{i} \bullet \underline{\hat{x}}_G \underline{\hat{k}}^{i} |}, \; \underline{\hat{x}}^{i}{}_{v2} = \underline{\hat{y}}^{i}{}_{v2} \times \underline{\hat{k}}^{i} \qquad (4)$$

and expand the incident $\underline{E}$ field after $\underline{\hat{x}}^{i}{}_{v1}$ and $\underline{\hat{x}}^{i}{}_{v2}$. This is illustrated in Fig. 6.

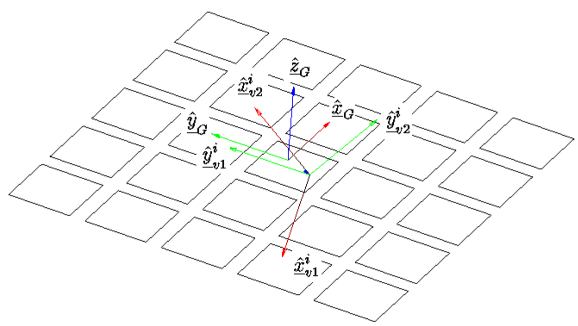

Figure 6. Skew angled coordinates for arbitrary incidence.

With the usual approximations, the field along $\underline{\hat{x}}^{i}{}_{v1}$ will only excite a field in the y_G directed slots and the field along $\underline{\hat{x}}^{i}{}_{v2}$ will only excite a field in the x_G directed slots. We can therefore separate the problem into two orthogonal strip arrays as before. It only remains to determine the position of the conducting walls below the strips.

This is considered in Fig. 7 for the $\underline{\hat{x}}^{i}{}_{v2}$ polarized field.

Let two rays in the incident wave hit the l.h.s. of the strip in P_1 and the r.h.s. in P_2. Two waves will be excited below the strip at an angle of ω_r determined by Snell's law. They meet in P_3 and if their phases do not agree, we adjust Δ accordingly. This determines the values of d_1 and d_2 and hence the position of the conducting wall.

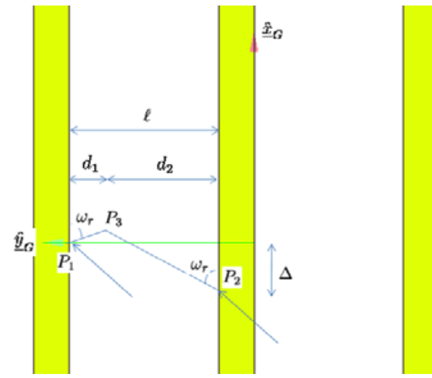

Figure 7. Determination of position of conducting walls.

The value of β can be expressed as in (2) with k_r replaced by $k_r \sin \omega_r$ and Ψ replaced by a value that depends on k_r , $\sin \omega_r$, d_1 , and d_2.
The same procedure is repeated for the other polarization.

5. CALCULATION OF RADIATION

We are now able to calculate the magnetic currents in all the gaps. Since each gap is short compared to the wavelength, we can assume that the currents have a constant amplitude and a progressing phase. As explained in the introduction, the current amplitude is doubled to account for the image and the radiated field is integrated in closed form using the free space Green's function. The far field from each gap can be written

$$\underline{H}_{1,n} = (1 + \beta_n)\underline{h}_n \qquad (5)$$

where β_n is the complex amplitude of the reflected wave from gap No. n, assuming that the field from the feed can be considered locally plane. Summing $\underline{H}_{1,n}$ over n leads to part of the reflected field to which must be added the reflection from the PEC surface supporting the currents. Suppose now that all β_n were zero, then all reflected fields, and hence the total field, would be zero. To achieve this we must add $\underline{H}_{2,n} = -\underline{h}_n$ to (5). The total field is thus obtained by replacing (1+ β_n) in (5) with β_n.
As an example consider the reflectarray in Fig.8.
It has 45 x 45 patches and is designed to radiate a maximum for (u,v)=(0.49,0.). The array is 45 x 45 cm, the frequency 10 GHz, and the feed is

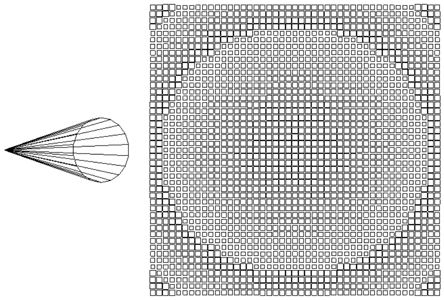

Figure 8. Reflectarray example..

a Gaussian feed, placed at (θ, φ)=(30,-180) deg., 60 cm from the centre of the array.
The u-v pattern for this example has been calculated by the described method, and the result is shown in Fig. 9.

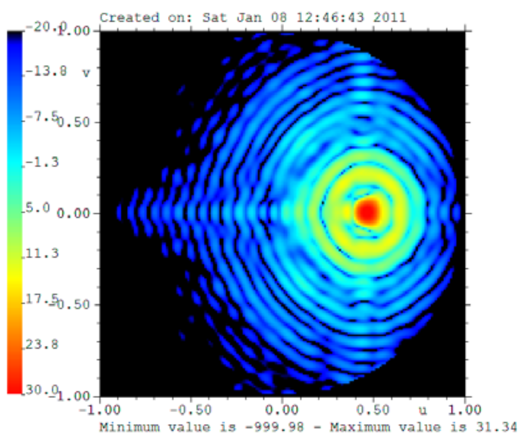

Figure 9 U-v pattern for reflectarray example.

If the maximum gain is optimized, using a non-linear least-squares optimizer, the patch pattern is changed only moderately as shown in Fig. 10. It is, however, possible to obtain an extra 1.0 dB gain and to suppress the sidelobes considerably.

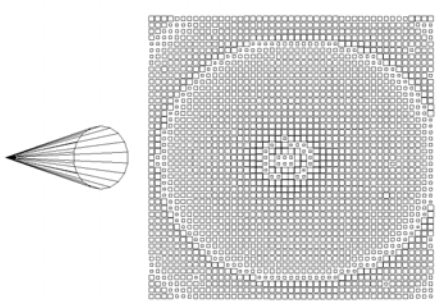

Figure 10. Optimized reflectarray example..

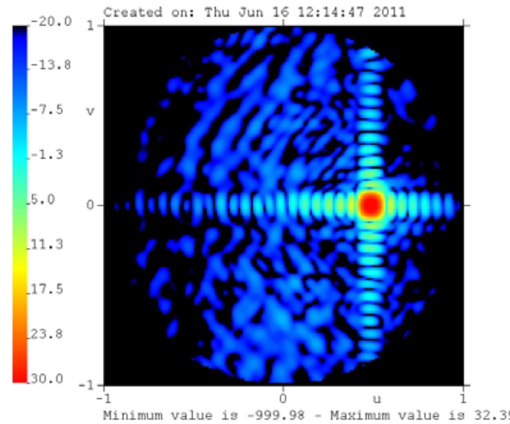

Figure 11 U-v pattern for optimized reflectarray example.

6. CONCLUSION

An intuitively simple method to analyze a certain class of reflectarrays has been presented. The method only relies on trivial mathematics and the result is obtained in closed form, making the method extremely fast, e.g. the above example was analyzed in 5.5 ms on a standard laptop. The complete expressions do not fit into the present format, but all details may be found in [1], which can be obtained from TICRA on request.
The method does not pretend to be as accurate as more sophisticated procedures, but is primarily intended for space mapping optimization. Also it provides a simple method to estimate general properties of an array. E.g. (1) makes it easy to estimate the effect of dielectric losses by simply adding an imaginary part to k_r and the procedure also explains why the optimum position for a circularly polarized feed is along a diagonal of the array.

7. REFERENCES

[1] Albertsen, N. Chr. (2010). *A simplified model of a reflectarray*, TICRA Report, WP1000 ESA Contract No. 4000101041.