Different Types of Circular Domain Wave-objects

M. Casaletti(1), S. Skokic(2), S. Maci* (1), and S. Sørensen(3)
(1) Department of Information Engineering University of Siena, Siena, Italy
(2) Faculty of Electrical Engineering and Computing, Unska 3, Zagreb, Croatia
(3) 3 TICRA, Læderstræde 34, Copenhagen K, Denmark
E-mail: casaletti@dii.unisi.it, sinisa.skokic@fer.hr, macis@dii.unisi.it, sbs@ticra.com.

Introduction

This paper discusses a novel method for computing aperture radiated fields by means of two new types of circular domain beams with azimuthal phase variation. These circular domain beams rigorously respect the wave equation and in their vector form, Maxwell's equations. The beam expansion of an aperture radiated field is simply obtained by first expanding the field spectrum in the aperture plane in terms of an angular Fourier series and next representing the radial spectral coefficients in terms of complex exponentials obtained by the Generalized Pencil-of-Function method (GPOF) [1] method. The starting radiation integral is this way reduced to a double sum of wave objects which can be computed analytically in both space and spectral domain. The vector form can be obtained through the use of vector potentials. This paper will only deal with the development and the propagation of the new wave objects. An example is presented to shows how the two different typologies of these new wave objects can be used to reconstruct the near and far field radiated by a circular apertures. The results are compared to those obtained via direct integration of the radiation integral.

Circular Domain Wave-Objects Formulation A

Let us consider the Fourier-type spectral radiation integral [2] in cylindrical coordinates:

$$I = \frac{1}{8\pi^2 j}\int_0^\infty\int_0^{2\pi} \tilde{g}\left(k_\rho,\alpha\right) e^{-j\rho k_\rho \cos(\alpha-\phi)} \frac{e^{-jz\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} k_\rho d\alpha dk_\rho \tag{1}$$

where $\tilde{g}\left(k_\rho,\alpha\right)$ denotes an aperture spectrum of either electric or magnetic field or of a scalar potential, while $e^{-jk\sqrt{k^2-k_\rho^2}} / \sqrt{k^2-k_\rho^2}$, is the spectral-domain representation of the free-space Green's function. Due to the inherent periodicity in α, $\tilde{g}\left(k_\rho,\alpha\right)$ can be expanded in a Fourier series

$$g\left(k_\rho,\alpha\right) = \sum_{n=-\infty}^{\infty} c_n\left(k_\rho\right) e^{-jn\alpha} \tag{2}$$

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and the integral in α in (1) can be evaluated in a closed form, yielding

$$I = \frac{(-j)^{n+1}}{4\pi} \sum_{n=-\infty}^{\infty} e^{-jn\phi} \int_{0}^{+\infty} c_n(k_\rho) \frac{e^{-jz\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} \mathrm{J}_n(\rho k_\rho) k_\rho dk_\rho \tag{3}$$

where J_n is the Bessel function of n-th order. The coefficients $c_n(k_\rho)$ are represented by using the Generalized Pencil of Function (GPOF) method [1] as

$$c_n(k_\rho) = \sum_{m=1}^{M} d_{mn} e^{b_{mn}\sqrt{k^2-k_\rho^2}} \tag{4}$$

where d_{mn} and b_{mn} are the output residues and poles of the GPOF algorithm. Note that $c_n(k_\rho)$ is represented as a function of $k_z = \sqrt{k^2-k_\rho^2}$. Using (4) in (3) leads to

$$I = \sum_{n}\sum_{m=1}^{M} a_{mn} W_n(\rho, \phi, z + jb_{mn}) \tag{5}$$

where $a_{mn} = (-j)^{n+1} d_{mn} / 4\pi$ and $W_n(\rho, \phi, \tilde{z})$ is defined by

$$W_n(\rho, \phi, \tilde{z}) = e^{-jn\phi} \int_{0}^{\infty} \frac{e^{-j\tilde{z}\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} J_n(\rho k_\rho) k_\rho dk_\rho \tag{6}$$

It is possible to find an analytical solution to the integral in (6). This is achieved by relating it to a similar integral whose closed-form solution is known [3] (Eq. 6.637). For n greater than -1, the following recurrence formula holds:

$$\begin{aligned} W_{n+1} = {} & e^{-j2\phi} W_{n-1} - 2e^{-j(n+1)\phi} (-j)^{n+1} \frac{k\pi}{8} \cos\tilde{\theta} \cdot \Big[-\mathrm{J}_{n/2-1}(d^-) \mathrm{H}^{(2)}_{n/2}(d^+) + \mathrm{J}_{n/2+1}(d^-) \mathrm{H}^{(2)}_{n/2}(d^+) \\ & + \mathrm{J}_{n/2}(d^-) \mathrm{H}^{(2)}_{n/2-1}(d^+) - \mathrm{J}_{n/2}(d^-) \mathrm{H}^{(2)}_{n/2+1}(d^+) \Big] \end{aligned} \tag{7}$$

where J_ν and $\mathrm{H}^{(2)}_\nu$ are respectively the Bessel function and the Hankel function of the second kind of order ν, evaluated at $d^- = -\frac{k}{2}(\tilde{r} - \tilde{z})$, $d^+ = \frac{k}{2}(\tilde{r} + \tilde{z})$.

Circular Domain Wave-Objects Formulation B

With the starting aperture field spectrum in (1) in mind, we multiply and divide the integrand by $(k_\rho/k)^{|n|}$. One of these two factors is included in the Fourier expansion yielding the coefficients $c_n(k_\rho)$ defined by (2). The subsequent GPOF expansion is *n*-dependent:

$$q_n(k_\rho) = c_n(k_\rho)\left(\frac{k}{k_\rho}\right)^{|n|} = \sum_{m=-M}^{M} d'_{mn}\, e^{b'_{mn}\sqrt{k^2-k_\rho^2}} \tag{8}$$

The initial radiation integral, as in the first case, reduces to the double sum:

$$I = \sum_n \sum_{m=1}^{M} a'_{mn} \Psi_n(\rho, \phi, jb'_{mn} + z) \tag{9}$$

where $a'_{mn} = (-j)^{n+1}\frac{1}{4\pi}d'_{mn}$, and the new wave objects Ψ_n assume a different form compared to (6), due to the presence of the term $\left(k_\rho/k\right)^{|n|}$ in the integrand:

$$\Psi_n(\rho,\phi,z) = e^{-jn\phi}\int_0^\infty \frac{e^{-jz\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} J_n(k_\rho \rho)\left(\frac{k_\rho}{k}\right)^{|n|} k_\rho\, dk_\rho. \tag{10}$$

A closed form solution to this integral can be constructed by using formulas in [3] (Eqs. 6.737.5-6.737.6), yielding

$$\Psi_n(\rho,\phi,\tilde{z}) = e^{-jn\phi} k \frac{\rho^n}{\left(\tilde{z}^2+\rho^2\right)^{n/2}} \sqrt{\frac{\pi}{2k\sqrt{\tilde{z}^2+\rho^2}}}\, \mathrm{H}^{(2)}_{n+1/2}\left(k\sqrt{\tilde{z}^2+\rho^2}\right) = e^{-jn\phi} k \sin^n\tilde{\theta}\, \mathrm{h}^{(2)}_n(k\tilde{r}) \tag{11}$$

where $\mathrm{h}^{(2)}_n(k\tilde{r})$ is the spherical Hankel function of the second kind.

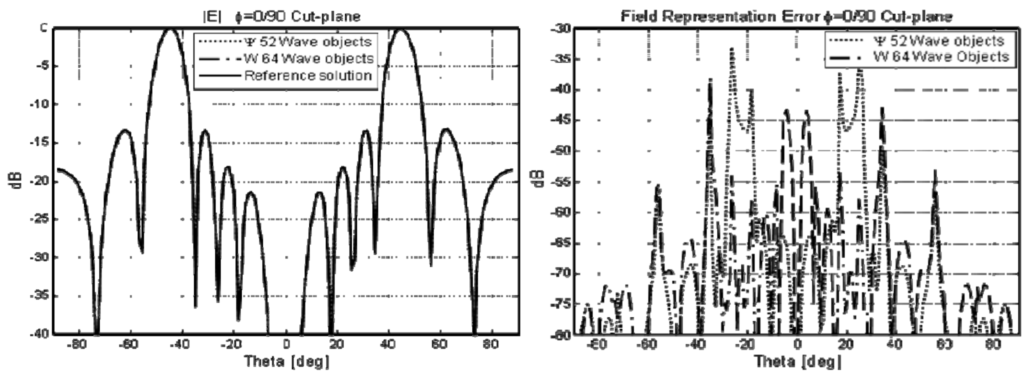
Numerical results

The described approach is applied here to calculate the radiation from a circular aperture. Let us consider a TM_{mn} circular waveguide mode defined by [4]:

$$\mathbf{E}_{mn}(\rho,\phi) = \alpha_{mn}\, \mathrm{J}'_m(\alpha_{mn}\rho)\cos(m\phi)\,\hat{\rho} - \frac{m}{\rho}\mathrm{J}_m(\alpha_{mn}\rho)\sin(m\phi)\,\hat{\phi} \tag{12}$$

where J_m and J'_m are the m-th order Bessel function and its derivative, respectively, and $\alpha_{mn} = \chi_{mn}/r_w$, where r_w is the radius of the circular waveguide (i.e. aperture), and χ_{mn} is the n-th zero of the m-th order Bessel function. This particular aperture field distribution has been selected because it possesses closed form expressions in both spectral and spatial domain, therefore allowing a simple construction of the reference solution. An aperture with radius $r_w = 4\lambda$ illuminated by a TM_{44} mode has been considered. For this aperture size, this particular mode is the closest one to cut-off, i.e., it possesses the eigenvalue α_{44} closest to the visible region boundary; this results in a fast aperture field variation. The spectrum is sampled only in the visible spectrum region.

The aperture field in (12) is described by 4 harmonics of the Fourier expansion (4). A total of 64 and 52 CBB's has been automatically generated by the GPOF procedure to represent the field in terms of W and Ψ, respectively.
Fig. 1a presents successful comparative results obtained using both formulations A and B; the reference solution is provided by the direct integration of (1). Fig. 1b and presents the absolute error between normalized fields. In particular, Fig.1a refers to the total electric field radiated in both ϕ =0° and ϕ =90° cut-planes sampled on a sphere of radius 300λ.



(a) (b)

Figure 1. a) E-plane (ϕ = 0) and H-plane (ϕ = 90º) cuts of the radiated total electric field magnitude by a TM_{44} circular waveguide mode, sampled on a sphere of radius 300λ; b) Corresponding field representation error.

References

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