Analytical Modelling of Waveguide Mode Launchers for Matched Feed Reflector Systems

Michael Forum Palvig[1,2] and Olav Breinbjerg[1]
[1]Department of Electrical Engineering
Technical University of Denmark
Kgs. Lyngby, Denmark
mpalvig@elektro.dtu.dk

Peter Meincke[2] and Erik Jørgensen[2]
[2]TICRA
Copenhagen, Denmark

***Abstract*—Matched feed horns aim to cancel cross polarization generated in offset reflector systems. An analytical method for predicting the mode spectrum generated by inclusions in such horns, e.g. stubs and pins, is presented. The theory is based on the reciprocity theorem with the inclusions represented by current sources. The model is supported by Method of Moments calculations in GRASP and very good agreement is seen. The model gives rise to many interesting observations and ideas for new or improved mode launchers for matched feeds.**

I. Introduction

Mode launchers are used in matched feeds for generating higher-order modes which create cross polarization cancelling that inherent to an offset reflector system. The invention was made by Rudge and Adatia [1] and has since received considerable attention in the literature (see e.g. [2], [3], [4]).

The mode launcher for matched feeds usually takes the form of pins attached to the wall of the waveguide or stubs/slots likewise along the wall. In circular symmetric horns, the mode launcher must exhibit non-circular symmetric geometry in order to excite the desired modes. Despite the mode launcher being a crucial part of the matched feed, there is very little literature explaining it's basic operation. Usually, the geometry of the mode launcher and the position of the inclusions are taken for granted and the motivation for the design is left out.

The exception is an analysis of radial pins using a Method of Moments approach with waveguide Green's functions [5]. In this paper, we develop a related, but simpler method for describing the operation of mode launchers. In essence, slots or stubs are modelled as magnetic current elements and pins are modelled as electric current elements. Using the reciprocity theorem and the orthogonality of the waveguide modes, it is possible to derive which modes are excited as well as the amplitude and phase relationships between them.

II. Model

Consider a waveguide with a source region containing electric and magnetic currents, $\mathbf{J}$ and $\mathbf{M}$. We do not assume a specific waveguide shape, only that we know an orthogonal eigenmode expansion of the fields. The fields from the current distribution can be found with the Green's functions associated with the waveguide shape, but since we are, in fact, not interested in the fields, we shall take the approach of [6, Sec. 4.10]. This method involves expanding the E- and H fields generated by the currents in waveguide eigenmodes and then successively using the reciprocity theorem with each mode of the expansion as the secondary field. Using field expansions as defined in [7] and including magnetic currents in the derivation, we arrive at the following expressions for mode coefficients generated by currents $\mathbf{J}$ and $\mathbf{M}$

$$C_i^{\pm} = \frac{Z_i}{2} \int_V \left(\mathbf{H}_i^{\mp} \cdot \mathbf{M} - \mathbf{E}_i^{\mp} \cdot \mathbf{J} \right) dV \tag{1}$$

where C_i^{+} and C_i^{-} are mode coefficients of the i-th generated mode travelling in the positive and negative z-direction, respectively. $\mathbf{E}_i^{\pm}$ and $\mathbf{H}_i^{\pm}$ are unity excited E- and H-fields of the i-th mode travelling in the positive ($^{+}$) and negative ($^{-}$) z-direction. Z_i is the wave impedance of the i-th mode. Thus, the excitation coefficient of a specific mode traveling in one direction can be found simply by projecting the current onto the field of the same mode travelling in the opposite direction.

The second part of the model is the excitation of said currents which is dependent on the incident mode. The current on the pin or in the stub aperture is proportional to the projection of the incident mode fields onto the current orientation at the current position:

$$\mathbf{J} \propto (\mathbf{E}_{inc} \cdot \hat{\mathbf{c}})\hat{\mathbf{c}} \tag{2}$$

$$\mathbf{M} \propto (\mathbf{H}_{inc} \cdot \hat{\mathbf{c}})\hat{\mathbf{c}}, \tag{3}$$

where $\hat{\mathbf{c}}$ is a unit vector in the direction of the current (e.g. the orientation of the pin). This is an accurate model for electrically small inclusions.

III. Application

Using the above model we examine a longitudinal stub on a circular waveguide of radius $a = 6.3\,\mathrm{mm}$ at $30\,\mathrm{GHz}$. The waveguide is $20\,\mathrm{mm}$ long and the stub is located in the middle, as seen in Fig. 1. We model the stub as a single infinitesimal magnetic current filament located at the stub aperture, oriented in the z-direction.

We consider the two orthogonal fundamental modes as incident, namely TE_{11}^{1} (x-polarized) and TE_{11}^{2} (y-polarized). The ϕ-position of the stub is varied. An accurate Method of Moments (MoM) simulation of the waveguide device in GRASP [8] serves to validate the method. Fig. 2 shows the results from the model (solid line) and the MoM simulation (stars). Since the model does not predict the absolute excitation of the currents, the model results are scaled by a single complex constant. The excitation relationship between the modes is predicted by the simple model and is in very good agreement with the simulated results as can be seen in the figure.

Note that the excited modes also have two orthogonal versions (except for modes with azimuthal index $m = 0$). For a traditional matched feed with the reflector offset in the yz-plane, two TE_{21} modes are needed, one for each polarization: TE_{21}^2 is needed for TE_{11}^1 operation (x-pol) and TE_{21}^1 is needed for TE_{11}^2 operation (y-pol) [9]. Thus, we want the purple curve in the top plot of Fig. 2 and the red curve in the bottom plot. It is easily deducible from the plots that this can be achieved by a stub at $90°$ and one at $35°$. It is also straight forward to see (when looking at the phase plots) that placing an extra stub at $145°$ will cancel the unwanted TE_{21}^1 contribution for x-pol (top plot), but add constructively for y-pol (bottom plot). However, these two stubs will deduct from the desired TE_{21}^2 generation for x-pol. We can move them to $45°$ and $135°$ and avoid this effect or move them still further to $50°$ and $130°$ which, assuming identical stubs, is the exact point that will ensure equal generation of TE_{21}^2 for x-pol and TE_{21}^1 for y-pol.

Many other observations can be made by examining these kinds of plots. The model is approximate, but gives valuable insight into the workings of existing mode launchers and can provide ideas for improved mode launcher designs. The model can also enter into more complex optimization procedures, as it is extremely fast to evaluate. Similar simulations made with radial pins modelled as ρ-directed electric currents are omitted here due to shortage of space.

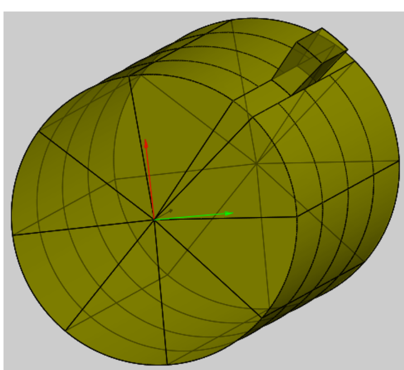

Fig. 1. MoM mesh of the mode launcher consisting of a longitudinal stub on a circular waveguide. In this picture the azimuthal position of the stub is $\phi = 45°$.

IV. Conclusion

A new model describing mode launcher operation for matched feeds is presented. It gives simple insight into previously presented designs and can form the basis of new and improved designs. An example with a longitudinal stub

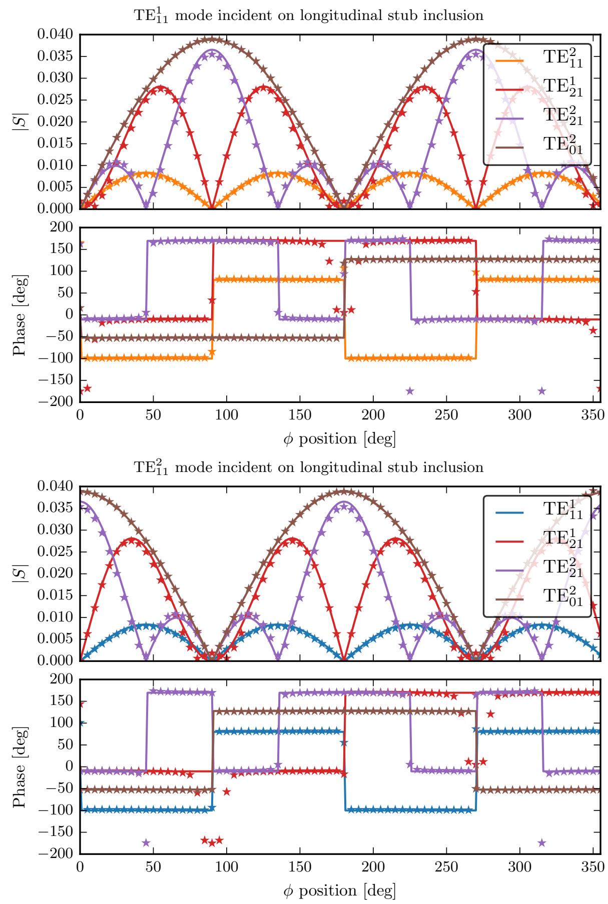

Fig. 2. Modes generated by longitudinal stub versus stub azimuthal position for x-polarized (top) and y-polarized (bottom) TE_{11} incidence. Solid lines are calculations from the simple model and the stars are full wave MoM simulations from GRASP.

inclusion in a circular waveguide shows excellent agreement with full-wave simulations.

References

[1] A. W. Rudge and N. A. Adatia, "New class of primary-feed antennas for use with offset parabolic-reflector antennas," *Electronics Letters*, vol. 11, no. 24, pp. 597–599, 1975.

[2] B. K. Watson, A. W. Rudge, and N. A. Adatia, "Dual-Polarised Mode Generator for Cross-Polar Ccmpensation in Offset Parabolic Reflector Antennas," *8th European Microwave Conference*, pp. 183–187, 1978.

[3] K. M. Prasad and L. Shafai, "Improving the symmetry of radiation patterns for offset reflectors illuminated by matched feeds," *IEEE Transactions on Antennas and Propagation*, vol. 36, no. 1, pp. 141–144, 1988.

[4] K. Bahadori and Y. Rahmat-Samii, "Tri-Mode Horn Feeds Revisited: Cross-Pol Reduction in Compact Offset Reflector Antennas," *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 9, pp. 2771–2775, 2009.

[5] R. Dey, S. Chakrabarty, R. Jyoti, and T. Kurian, "Higher Order Mode Analysis of Dual-Post Discontinuity in a Circular Waveguide," *IETE Journal of Research*, pp. 1–8, 2015.

[6] R. E. Collin, *Foundations for microwave engineering*. McGraw-Hill, 1992.

[7] N. Marcuvitz, *Waveguide handbook*. Peregrinus, 1986.

[8] "GRASP Software," TICRA, Copenhagen, Denmark, *www.ticra.com*.

[9] M. F. Palvig, E. Jørgensen, P. Meincke, and O. Breinbjerg, "Optimization Procedure for Wideband Matched Feed Design," in *To be presented at the 2016 European Conference on Antennas and Propagation*, 2016.